Prince Sultan University

Department of Mathematical Sciences

Final Exam

Semester II, 2005 Spring (042)

13th June, 2005

MATH 111 – CALCULUS I

Mr. Khaled Naseralla

Time Allowed : 3 hours
Maximum Points: 100 points

Name of the student : _____

ID number : _____

Section : _____

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 8 pages and 16 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	*Your Score*
Q.1	4	
Q.2	5	
Q.3	4	
Q.4	4	
Q.5	15	
Q.6	6	
Q.7	4	
Q.8	15	
Q.9	5	
Q.10	5	
Q.11	5	
Q.12	6	
Q.13	8	
Q.14	5	
Q.15	5	
Q.16	4	
Total	**100**	

Q.1: **Find the natural domain of** $f(x)=\sqrt{9-4x^2}$ **.** **(6 points)**

Q.2: **Let** $f(x)=\begin{cases}(x+c)^2 & x<3\\ 5x+c & x\geq 3\end{cases}$ **(8 points)**

Find the value(s) of c **such that** $f(x)$ **is continuous everywhere.**

Q.3: **Given that** $f(x)=\sqrt{x-1}$ **and** $g(x)=x^2$ **find:** **(8 points)**

a) $(f\circ g)(x)$

b) $(g\circ f)(2)$

Q.4: **Find an equation of the family of lines that pass through the intersection** **(8 points)**

of $2x+3y=5$ **and** $4x+2y=-2$

Q.5: **Find the limits:** **(8 points)**

a) $\lim\limits_{x\to 2}\dfrac{3x^2-x-10}{x^2-4}$

b) $\lim\limits_{x\to 4}\dfrac{3-\sqrt{x+5}}{4-x}$

c) $\lim\limits_{\theta\to 0}\dfrac{1-\cos\theta}{\theta^2}$

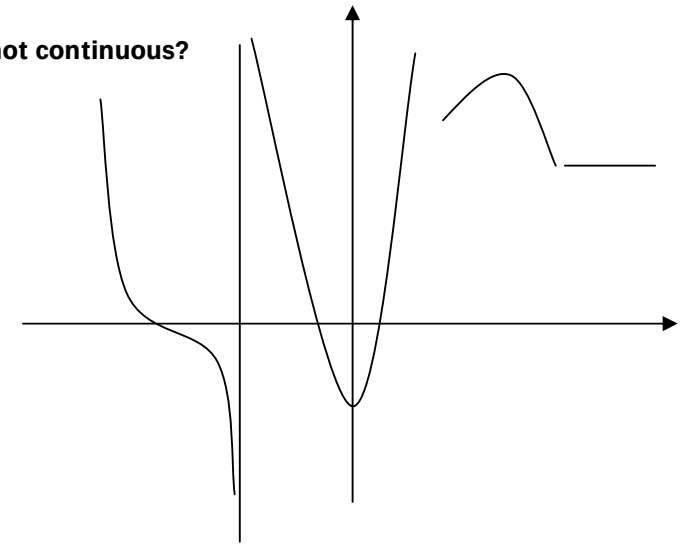
d) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{4x^2+5}}{6x-7}$

e) $\lim\limits_{x\to\infty}\dfrac{2+6x^2+5x^4}{7x+10x^4}$

Q.6: **Consider the function f in the accompanying figure.** **(10 points)**

a) For what value of x , $\lim f(x)$ does not exist ?

b) For what value of x , the function f is not continuous? Give the reason.



Q.7: **Find the slope of the tangent to the graph 0f $x^2y^7 - x^3y^2 = 2$ at $(-1,1)$** **(8 points)**

Then find the equation of the tangent line.

Q.8 **: Find the derivative of the following. Simplify when possible** **(12 points)**

a) $f(x) = 5^x e^{6x}$

b) $y = \cos^5(\tan 4x)$

c) $y = \dfrac{1+\tan^{-1}x}{2-3\tan^{-1}x}$

d) $y = \dfrac{e^x - e^{-x}}{e^x}$

e) $y = (x^4+3)^{\cos x}$

f) $f(x) = \ln\left(\dfrac{\sin x}{\sin x + \cos x}\right)$

Q.9: **A spherical snowball is melting in such a way that the radius is decreasing at a rate of 1.1 m/min. At what rate is the volume decreasing when the radius is 3m?** **(12 points)**

Q.10: **Find** dy **and** Δy **for** $f(x) = x^3 + x^2 - 1$ **if** x **changes from** 2 **to** 2.1 **Then find the average rate of change.** **(12 points)**

Q.11: **Find the absolute maximum and absolute minimum values of** f **on the given closed interval, and state where these values occur.** **(12 points)**

$f(x) = 2x^3 - 3x^2 - 12x + 1$ **in** $[-2, 3]$

Q.12: **The position function of a particle moving along a coordinate line is given by** $s(t) = t^3 - 3t^2 + 4$ **,** $t \geq 0$ **where** s **is in meters(m) and** t **is in seconds(s)** **(12 points)**

a) **At what time is the particle stopped?**
b) **When is the particle speeding up? Slowing down?**

Q.13: **Let** $f(x) = 2x^3 + 9x^2 + 12x + 2$ **, find** **(12 points)**

a) **The intervals on which** $f(x)$ **is increasing.**
b) **The intervals on which** $f(x)$ **is decreasing.**
c) **The intervals on which** $f(x)$ **is concave up.**
d) **The intervals on which** $f(x)$ **is concave down.**
e) **The local extrema of** $f(x)$ **and classify them as maximum or minimum.**

Q.14: **A rectangular box with an open top and square base is being made from metal sheet. There is** $768\ in^2$ **of metal available. Find the dimensions of the box that has the maximum volume.** **(12 points)**

Q.15: **Give a complete graph of the polynomial** $f(x) = x^3 - 3x^2$ **. Find and locate the relative extrema and the inflection points.** **(12 points)**

Q.16: **Let** $f(x) = ax^2 + bx + 8$ **,** $a \neq 0$ **. If** f **has a local minimum of** 6 **at** $x = 2$ **. Find** a **and** b **(12 points)**