Prince Sultan University

Department of Mathematics
and
General Sciences

Math 225
Final Exam Term 162 May 13, 2017
Duration: 180 minutes

Name: Section 429, 666
Student Number:

Grading policy:

Questions	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9	Q.10	Total
Question Mark	10	8	6	7	6	10	10	7	13	13	90
Student Mark											

40

Good Luck

Q.1 (10 points) Solve the nonlinear differential equation $y' + \left(\frac{1}{x}\right) y = (xlnx)y^2$.

Q.2 (8 points) Find the general solution of Euler equation $x^2y'' + 5xy' + 3y = 0$ and describe how the solution behaves as $x \to 0$.

Q.3 (6 points) Use the definition to find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t < 3 \\ 0, & 3 \le t < \infty \end{cases}$.

Q.4 (7 points) Find the inverse Laplace transform of $F(s) = \frac{2s-5}{s^2+2s+10}$.

Q.5 (6 points) Show that the functions $f(x) = e^x$ and $g(x) = xe^{-x} - e^{-x}$ are orthogonal on $[0,2]$.

Q.6 (10 points) Use Laplace transformation method to solve the initial value problem $y'' + 4y' + 4y = g(t), y(0) = 1, y'(0) = -2$.

Q.7 (10 points) Find the general solution $y_g = y_h + Y_p$ of the equation

$$y^{(4)} + 2y''' + 2y'' = 3e^t + 2te^{-t} + e^{-t}sint$$

Do not evaluate the constants.

Q.8 (7 points) Verify that the functions $1,\ x,\ x^3$ are solutions of the differential equation $xy''' - y'' = 0$ and determine their Wronskian.

Q.9 (13 points) Consider the equation $4xy'' + y' + xy = 0$.

a. Show that $x_0 = 0$ is a regular singular point.
b. Find the indicial equation, the recurrence relation and the roots of the indicial equation.
c. Find a series solution corresponding to the largest root.

Q.10 (13 points) Solve the heat equation:

$$2u_{xx} = u_t,\ 0 < x < 6, t > 0$$

subject to the conditions: $u(0,t) = 0$, $u(6,t) = 0$, and $u(x,0) = x(6-x)$ where the rod is assumed to be of length 6.