



Prince Sultan University

Department of Mathematical Sciences

Semester II, 2012 SPRING (Term 112)
May 27, 2012

MATH 111 – Calculus I

Final Exam

Time Allowed : 120 minutes

Maximum Points : 100 points

Name of the student: _____

ID number : _____

Section : _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination **has 9 problems**, some with several parts and a total of 7 pages. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1	20	
Q.2 , Q.3	15	
Q.4 , Q.5	13	
Q.6	20	
Q7 , Q.8	14	
Q.9	18	
Total	100	

Q.1 (20 points): Find the derivative, $\frac{dy}{dx}$. (Simplify as much as possible)

(i) $y = 3^x \ln(1 + x^2)$

(ii) $y = \sin^{-1}(1 + x^2)$

(iii) $y = \frac{\sqrt{x \sin x}}{1 + e^{1-2x}}$

(iv) $y = x^{\sin(3x)}$

(iv) $y = \tanh^{-1}\left(\frac{2\pi}{x}\right)$

Q.2 (7 points): Let $f(x) = x|x-1|$. Discuss the differentiability of f .

Q.3 (8 points): For $y = f(x)$ such that $xe^{xy} = y + \sin^2(x) + 2$, find the equation of the tangent line to the graph of f at $x = 0$.

Choose one of the two questions to solve

Q.4 (6 points): Prove that the equation $e^{x-1} + x + 2 = 0$ has exactly one real root.

OR

Q.4 (6 points): Let f be a differentiable function on $(-\infty, \infty)$. If

$$f(1) = -2, \quad f'(x) \geq 2, \quad \text{for every } x \in [1, 6].$$

Show that $f(6) \geq 8$. (Hint: Use Mean Value Theorem)

Q.5 (7 points): Find the value of the constant k such that the function g

is **continuous everywhere**
$$g(x) = \begin{cases} \frac{\sin(kx)}{2x} & \text{if } x \neq 0 \\ 7 & \text{if } x = 0 \end{cases}$$

Q.6 (20 points): Evaluate the following limits:

(i) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 3x + 2}$

(ii) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{-3 + \sqrt{10 - x}}$

(iii) $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$

(iv) $\lim_{x \rightarrow 0} \frac{2x - |x|}{|3x| - 2x}$

(v) $\lim_{x \rightarrow +\infty} \left(\frac{x}{2+x} \right)^{3x}$

Q.7 (7 points): A cylindrical can, open at the top, is to hold 600 cm^3 of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.

Q.8 (7 points): A ladder 10 m long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at rate of 2 m/sec., how fast is the ladder sliding down the building when the top of the ladder is 5 m above the ground?

Q.9 (18 points): Consider the function $f(x) = \frac{2x^2}{x^2 - 1}$.

- (a) Find the horizontal and vertical asymptotes of f , if they exist.
- (b) Find critical number(s) for f and the intervals where f is increasing or decreasing.
- (c) Find the local extrema of f , if any.
- (d) Find the inflection point(s) of f and the intervals where f is concave up or down.
- (e) Sketch the graph of f showing all significant features.

