Prince Sultan University
Numerical Analysis: MATH 221
Final Exam

Semester I, Term 151
Dr. Ahmed Kaffel
Wednesday, December 30 , 2015
Time Allowed: **2 hours**

Student Name: _____

Student ID #: _____ Section #: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 4 questions, the last one is a 3 points bonus. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1	16	
2	16	
3,4	8,3	
Total	**40**	

Question 1. [16 points]

a) **[6 points]** Let $f(x) = 3x^2 + e^x$. Find the Newton's formula and then use it to compute a solution accurate to within 10^{-4} for $f(x) = 0$, starting with $x_0 = 4$.

b) **[4 points]** **Use** approximations to the fixed point iteration method to find the first four the root of $x_{n+1} = \frac{1}{2} Ln(1 + x_n), \quad n \geq 0,$ using $x_0 = 4$.

c) **[6 points]** Show that the equation $x^3 - 3x - \frac{1}{2} = 0$ has a root in the interval $[1, 2]$. Estimate the number of the required iterations needed to achieve an approximation of this root to an accuracy of 10^{-5} using the bisection method.

Question 2. [16 points]

a) **[5 points]** Determine the value of step size h and the number of subintervals n to approximate the integral $I = \int_0^2 \frac{dx}{4+x}$ to an accuracy of 10^{-5}, then compute the approximation, using Simpson's rule.

b) **[5 points]** Let $f(x) = 1 + e^{2x}$. Use the central formula with h=0.01 to approximate the value of $f''(2.01)$. Then use the forward formula to approximate the value of $f'(3.01)$. Then find the actual errors for your approximations.

c) **[6 points]** Use the Runge-Kutta method of order 4 on the below initial value problem to approximate y(0.1):

$$yy'+2xy = x+1, \quad y(0) = 2, \quad n = 1.$$

Then compute the error bound for your approximation.

Question 3. [8 points]

Let $f(x) = \dfrac{e^{2x}}{x+2}$.

a) **[3 points]** Find the third Newton's Divided Difference $f[x_0, x_1, x_2]$ of the function f relative to $x_0 = 1,\ x_1 = 2,\ x_2 = 3.$

b) **[5 points]** Use the Lagrange formula based on the points $x_0 = 1, x_1 = 2$, and $x_2 = 3$ to find the quadratic polynomial $p_2(x)$ to approximate the function $f(x)$ at $x_2 = 1.3$. Also compute the error bound for your approximation.

Question 4. (3 Bonus points)

We consider the following machine number

01 0000000010 10110110001000000000…….00

Find the decimal number that represents this machine number.