Prince Sultan University

Math 211
Final Exam
First Semester, Term 141
Saturday, January 3, 2014

Time Allowed: 120 minutes (2 hours)

Student Name:_____

Student ID #: _____

Important Instructions:

1.You may use a scientific calculator that does not have programming or graphing capabilities.
2.You may NOT borrow a calculator from anyone.
3.You may NOT use notes or any textbook.
4.Talking during the examination is NOT allowed.
5.Your exam will be taken immediately if your mobile phone is seen or heard.
6.Looking around or making an attempt to cheat will result in your exam being cancelled.

Problems	Max points	Student's Points
1,2,3	18	
4,5,6	16	
7,8,9	20	
10	12	
11,12	14	
Total	**80**	
Total	**40**	

1) [8 points] Given the function $f(x) = x^3 + 6x^2 + 9x$

a) Find the intervals of increase and decrease.

b) Determine the coordinates of the relative maxima or minima.

c) Determine intervals of concavity and find the inflection points. **DO NOT GRAPH.**

2) [6 points] Find the absolute maximum and minimum (if any) for the function $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 5$ over the interval $-2 \le x \le 3$.

3) [4 points] Given that $y = 2xe^{x^2}$, show that the equation of the tangent line at $x = 1$ is given by $y = 6ex - 4e$

4) [6 points] A retailer estimates that when x units of a particular commodity are sold the revenue generated will be $R(x)=125+12\ln\left(5x^2-2\right)$ dollars. Use *marginal revenue analysis* to estimate the additional revenue derived from selling the 5^{th} unit.

5) [4 points] Use logarithmic differentiation to find the first derivative of $y=\dfrac{(x+2)^2\ \sqrt{x}}{e^3\left(2x^2-1\right)}$

6) [6 points] Find the first derivative of the following:

a) $x^3+y^3=6xy$

b) $y=\dfrac{5}{e^x+x}$

7) [6 points] A manufacturer supplies $S(p) = 0.5p^2 + 3p + 7$ hundred units of a particular commodity to the market when the price is *p* dollars per units. Find the **<u>average supply</u>** as the price varies from $p = \$2$ to $p = \$5$.

8) [6 points] Find the area of the region bounded by the curve $y = x^2 - 6x + 8$ and the curve $y = -x^2 + 4x$

9) [8 points] A manufacturer has found that the marginal cost is $C'(x) = \dfrac{9x^2 + 15x}{\sqrt{2x^3 + 5x^2}}$ dollars per unit when the level of production is *x* units. The cost of producing 10 units is \$200.

a) Find the Cost function $C(x)$

b) What is the cost of producing 20 units?

10) [12 points] Find the following integrals:

a) $\int_1^4 \left(\frac{2}{x} + \frac{3}{2}\sqrt{x} + 3 \right) dx$

b) $\int_0^1 (x+2)\, e^{x^2+4x+1}\, dx$ (use u substitution)

c) $\int x^2 e^{-x/2}\, dx$

11) [6 points] Given that $f(x,y) = 2x^2\sqrt{y} + e^{xy}$ find the following:

a) The partial derivatives f_x and f_y

b) The domain of $f(x,y)$

12) [8 points] Suppose that at a certain factory, output is given by the *Cobb-Douglas* production function $Q(K,L) = 30K^{0.3}L^{0.7}$ units, where K is the capital investment measured in units of \$1,000 and L the size of the labor force measured in worker-hours.

a) Compute the output if the capital investment is \$115,000 and the size labor force is 900 worker hours.

b) What happens to the output if the capital investment and the labor force is **tripled**?

c) Find the **marginal productivity of capital** Q_K when the capital investment is \$150,000 and the size labor force is 1100 worker hours.