Prince Sultan University
Department of Mathematical Sciences

Semester II, 2013 SPRING (Term 122)
May 25, 2013

MATH 111 – Calculus I
Final Exam

Time Allowed : 120 minutes
Maximum Points : 100 points

Name of the student: ____

ID number : ______________________________

Dr. Younis Zaidan	Dr. Mohamed Siddique	Mr. Khaled Naseralla
Section 223	Section 224	Section 225
10 ---- 11	11 ----- 12	8 ----- 9

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination **has 12 problems** , some with several parts and a total of 8 pages. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1	16	
Q.2 , Q.3 , Q.4	15	
Q.5 , Q.6	13	
Q.7 , Q.8	12	
Q.9	16	
Q.10 , Q.11	12	
Q.12	16	
Total	100	

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Q.1 (16 points): Find the derivative, $\frac{dy}{dx}$. **(Simplify as much as possible)**

(i) $y = \tan\left(\frac{t}{1+t^2}\right)^2$

(ii) $y = \frac{\sqrt{x+1}.(2-x)^5}{(x+3)^7}$

(iii) $y = e^{\cos(x)} + \cos(e^{3x})$

(iv) $y = \sqrt{x}\,\sin^{-1}\left(\sqrt{x}\right)$

<u>Q.2 (5 points):</u> Find $\frac{dy}{dx}$: $x + y\sqrt{1+2x} = 2x\cos(y)$

<u>Q.3 (5 points):</u> At what point(s) on the curve $y = [\ln(x+4)]^2$ is the tangent horizontal?

<u>Q.4 (5 points):</u> If $f(x)$ and $g(x)$ are differentiable functions at $x = 2$ such that $f(2) = 3$, $f'(2) = -1$, $h(2) = 2$, $h'(2) = -1$, and $g(x) = xf(x) + \frac{f(x)}{h(x)}$. Find $g'(2)$

Q.5 (8 points): Let $f(x)=\begin{cases} x^2-1 & \text{if } x\le 1 \\ k(x-1) & \text{if } x>1 \end{cases}$

(i) For what values of k is $f(x)$ continuous?

(ii) For what values of k is $f(x)$ differentiable?

Q.6 (5 points): Suppose that $3\le f'(x)\le 5$ for all values of x. Show that $18\le f(8)-f(2)\le 30$ by using the Mean Value Theorem.

Q.7 (6 points): Find the absolute maximum and absolute minimum values of the function $f(x) = \dfrac{x}{x^2+1}$ on the interval [0, 2].

Q.8 (6 points): Let $f(x) = x^4(x-1)^3$.

(a) Find the critical numbers of f, if any.

(b) Find the local minimum and local maximum values of f, if any.

Q.9 (16 points): Evaluate the following limits:

a) $\lim\limits_{x\to 1}\dfrac{2x^2+3x-5}{x-1}$

b) $\lim\limits_{t\to 0}\dfrac{\cos t}{t}$

c) $\lim\limits_{x\to 0} x^4\cos\left(\dfrac{3}{x}\right)$

d) $\lim\limits_{x\to 0}\dfrac{2x^9-\sin^9 x}{5x^9}$

Q.10 (6 points): Find the following limit by using L'Hospital's Rule: $\lim_{x \to \infty}\left(1+\frac{a}{x}\right)^{bx}$ where a and b are constant numbers.

Q.11 (6 points): A box with a square base and open top must have a volume of 4,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

Q.12 (16 points): Let $f(x) = \dfrac{x^2+1}{x^2-4}$.

a) Find the domain.
b) Find the *x* and *y*-intercepts.
c) Find the vertical and horizontal Asymptotes.
d) Find the critical points numbers.
e) Find the intervals on which *f* is increasing and the intervals on which *f* is decreasing.
f) Find the local minimum and local maximum values, if any.
g) Find the intervals on which *f* is concave up and the intervals on which *f* is concave down.
h) Find the inflection points.
i) Sketch the graph of *f*. (Show asymptotes).