Prince Sultan University

Deanship of Educational Services
Department of Mathematics
and
General Sciences

COURSE DETAILS:

DIFFERENTIAL EQUATIONS	MATH 225	FINAL EXAM
Semester:	Spring Semester --Term 172	
Date:	Saturday, May 05, 2018	
Time Allowed:	180 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	640, 159
Instructor's Name:	J. Alzabut

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 2	Page 3	Page 4	Page 5	Page 6	Total
Questions	**Q.1 (12 pts)**	**Q.2, Q.3 (18 pts)**	**Q.4 (16 pts)**	**Q.5, Q.6 (18 pts)**	**Q.7, Q.8 (16 pts)**	**80**
Student Marks						

Q.1 (4+8 points) Consider the equation

$$x\frac{dy}{dx} + y = 2x^4 y^4 \qquad \text{(I)}$$

a) Show that the substitution $v = y^{-3}$ transforms equation (I) into the differential equation

$$\frac{dv}{dx} - \frac{3}{x}v = -6x^3. \qquad \text{(II)}$$

b) Solve equation (II) and find a particular solution of equation (I) with $y(1) = 1$ in the form $y^3 = f(x)$.

Q.2 (8 points) Show that the functions $y_1(x) = \sin 4x$ and $y_2(x) = \cos 4x$ are solutions of the equation $y'' + 16y = 0$ and form a fundamental set of solutions.

Q.3 (10 points) Solve the equation $xy^3 dx + \left(x^2 y^2 + 1\right) dy = 0.$

Q.4 (7+9 points) Consider the IVP $y'' + y' = 2e^x,\ y(0) = 0,\ y'(0) = 1$

a) Find the general solution of IVP by using method of undetermined coefficients.

b) Find the general solution of the IVP by using method of Laplace transform.

Q.5 (7 points) Decide whether or not the point $x_0 = 0$ is a regular singular point for the equation $x^2 y'' + 2\left(e^x - 1\right)y' + e^{-x}\left(\cos x\right)y = 0.$ Explain your answer.

Q.6 (11 points) Use power series to solve the differential equation $y'' - xy' - y = 0$ around the ordinary point $x_0 = 0.$

Q.7 (7 points) Compute the Fourier sine series of $f(x)=1$ on $0\le x\le 1$. Find the coefficients explicitly and sketch its graph backward and forward for $-2\le x\le 4$.

Q.8 (9 points) Find the product solutions $u(x,t)=X(x).T(t)$ of the heat equation $u_t=4u_{xx}$.