Prince Sultan University

Math 113
Final Exam
Second Semester, Term 122
Tuesday, May 28, 2013

Time Allowed: 150 minutes

Student Name: _____

Student ID #: _____

Section Number: _____

Instructor's Name: Dr. Abdulwahed Hamdi, Dr. Aiman Mukheimer

Important Instructions:
1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. Talking during the examination is NOT allowed.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 15 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1	12	
2,3,4	10	
5,6,7	10	
8	12	
9	12	
10,11,12	10	
13	12	
14	12	
15	10	
Total	100	

1. (12 points) Evaluate each of the following integrals:
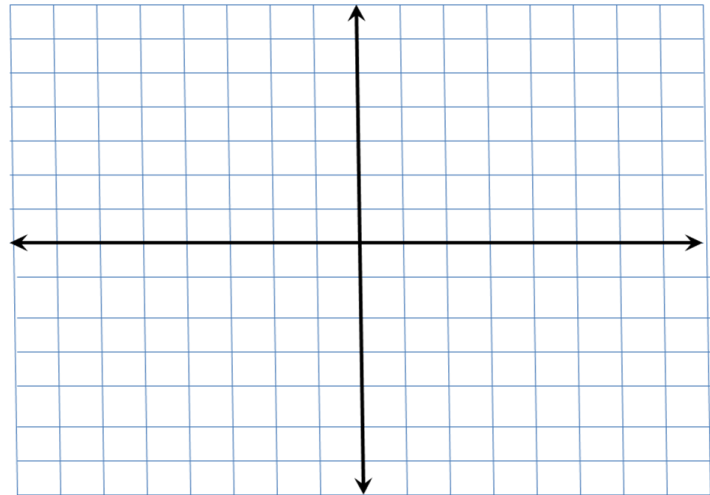
a) $\int \dfrac{x^2 e^{x^3}}{e^{x^3}+1}dx$

b) $\int \dfrac{1}{x(9+(\ln x)^2)}dx$

c) $\int \dfrac{2e^x + e^{2x}}{4+e^{2x}}dx$

d) $\int \dfrac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}dx$

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2. (3 points) If $f(x) = \int_{0}^{g(x)} \frac{1}{\sqrt{1+t^4}} dt$, where $g(x) = \int_{0}^{\cos x} [1+\sin(t^4)]dt$, find $f'(\frac{\pi}{2})$.

3. (3 points) Sketch the region bounded by the curves $y = \sqrt{25 - x^2}$ $y = 0, x = 2$ *and* $x = 4$ and **set up only** the integral that Consider the volume generated when the region with the given boundaries is rotated about the x-axis.

4. (4 points) Find the volume that results when revolving the area contained between $y = -x^2 + 2x + 1$; $y = 1$ about $x = -1$
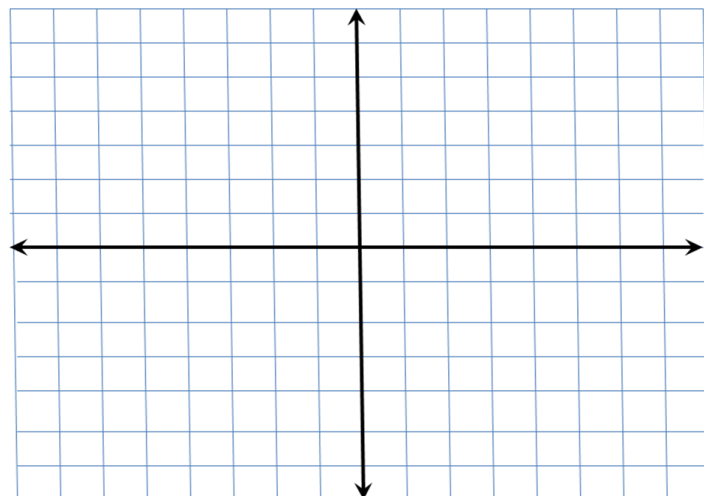
5. (4 points) Find the exact length of the curve $y = \ln(\cos x)$, $0 \le x \le \dfrac{\pi}{3}$.

6. (3 points) Prove the reduction formula: $\displaystyle\int \cos^n x \, dx = \dfrac{1}{n} \cdot \cos^{n-1} x \cdot \sin x + \dfrac{n-1}{n} \cdot \int \cos^{n-2} x \, dx$

7. (3 points) Find the average value of the function: $f(x) = \sin^2 x \cos^3 x$ on the interval $[-\pi, \pi]$.

8. (12 points) Evaluate each of the following integrals:

a) $\int \dfrac{3 - 2\tan x}{2 + 3\tan x} dx$

b) $\int \dfrac{3x^2 - x + 1}{x^3 - x^2} dx$

c) $\int_0^{\pi/6} x \sin^2(3x)\, dx$

d) $\int_1^{\sqrt{3}} \dfrac{\sqrt{1 + x^2}}{x^2}\, dx$

9. (12 points) Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent.

a) $\displaystyle\int_{2}^{3} \frac{1}{\sqrt{3-x}} dx$

b) $\displaystyle\int_{0}^{\pi} \left(\frac{1+2\cos x}{x+2\sin x} - \frac{1}{x}\right) dx$

c) $\displaystyle\int_{1}^{\infty} e^{x^2+x+1} dx$

10.(3 points) Determine whether the sequence $\left\{ (1+\dfrac{3}{n})^{4n} \right\}_{n=1}^{\infty}$ converges or diverges. If it converges, find its limit.

11.(4 points) Find the sum of the series: $\displaystyle\sum_{n=1}^{\infty}[\tan^{-1}(n+1)-\tan^{-1}(n)]$

12.(3 points) If the nth term of the sequence of the partial sum of a convergent series $\displaystyle\sum_{n=1}^{\infty}a_n$ is given by $s_n = 3-n2^{-n}$. Find a_n **and** $\displaystyle\sum_{n=1}^{\infty}a_n$.

13.(12 points) Determine whether the following series converges or diverges. **Justify your answers in details.**

a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{3n}{8n+3}\right)$

b) $\displaystyle\sum_{k=0}^{\infty} \frac{1+\sin k}{10^k}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$

14.(12 points) Determine whether the following series are absolutely convergent, conditionally convergent or divergent. **Justify your answers in details.**

a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-3)^{2k}}{k^2 7^k}$

b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{3n}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2)^{n^2}}{(2n+1)!}$

15.(10 points) Find the radius and the interval of convergence of the following series:
Justify your answers in details.

a) $\sum_{n=1}^{\infty} \dfrac{2^n (x-2)^n}{(n+2)!}$

b) $\sum_{n=2}^{\infty} (-1)^n \dfrac{x^n}{4^n \ln n}$