Prince Sultan University

Department of Mathematics and Physical Sciences

Math 223
Final Examination
Semester I, Term 101
Saturday, January 22, 2011

Time Allowed: 120 minutes

Name:

Student Number:

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 9 problems, some with several parts. Make sure your paper has all these problems.

Problems	**Max points**	**Student's Points**
1	30	
2,3,4	21	
5,6	16	
7	15	
8,9	18	
Total	**100**	

Question.1 (2 points each) Write True(T) or False(F) for each of the following statements:

	Statements	True(T) or False(F)
1.	The two planes $2y = 8x - 4z + 5$ and $x = \frac{1}{2}z + \frac{1}{4}y$ are parallel.	
2.	The distance between $P_1(3,4)$ and $P_2(5,7)$ is 4.	
3.	If A and B are square matrices of the same size, then $\det(A+B) = \det(A) + \det(B)$	
4.	If A is any matrix, then $rank(A) = rank(A^T)$.	
5.	A homogenous system of linear equations with more unknowns than equations has infinitely many solutions.	
6.	If kA is invertible matrix, then $(kA)^{-1} = kA^{-1}$.	
7.	The cosine of the angle between $u = (1,-3)$ and $v = (2,4)$ is 42°.	
8.	$\begin{vmatrix} a & b & c \\ d & e & f \\ g & i & h \end{vmatrix} = -\begin{vmatrix} g & i & h \\ d & e & f \\ a & b & c \end{vmatrix}$	
9.	The vectors $v_1 = (-1,2,6)$, $v_2 = (3,-2,4)$ and $v_3 = (-3,6,18)$ span R^3.	
10.	If $\{v_1, v_2, v_3\}$ is linearly independent set, then so is the set $\{kv_1, kv_2, kv_3\}$ for every nonzero scalar k.	
11.	If A is an $n \times n$ matrix and if $T_A : R^n \to R^n$ is multiplication by A. If T_A is one—to—one then A is expressible as a product of elementary matrices.	
12.	If an $n \times n$ matrix A has n distinct eigenvalues, then A is not diagonalizable.	
13.	The solution of $y' = ay$ is $y = ce^{ax}$, $a, c \in R$.	
14.	A square matrix A is invertible if and only if $\lambda = 0$ is an eigenvalue of A.	
15.	Let A be an $n \times n$ matrix. If A is ivertible, then $Ax = 0$ has only the trivial solution.	

Question.2 (8 points)(3.4) Let $u = (-1,2,4)$ and $v = (1,-1,3)$.

a) Find the vector component of u along v $\left(\Pr oj_v u\right)$.

b) If $w = (-1,1,5)$. Find the scalar triple product $u \bullet (v \times w)$.

Question.3 (7 points)(6.1)

Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$. Show that $\langle u, v \rangle = 4u_1v_1 + u_2v_1 + u_1v_2 + 4u_2v_2$ is an inner product on R^2.

Question.4 (6 points)(3.5)

Find an equation for the plane passing through the given points $P(5,4,3), Q(4,3,1), R(1,5,4)$.

<u>Question.5 (7 points) (8.3)</u> Let $T: R^2 \to R^2$ be defined by $T(x, y) = (2x + 3y, x - y)$. Determine whether the linear transformation is one—to—one. If so, find $T^{-1}(x, y)$.

<u>Question.6 (9 points)(8.1)</u> Consider the basis $S = \{v_1, v_2\}$ for R^2 where $v_1 = (1,1)$ and $v_2 = (3,-1)$. Let $T: R^2 \to R^2$ be the linear operator such that $T(v_1) = (1,-2)$ and $T(v_2) = (-4,1)$. Find a formula for $T(x_1, x_2)$. Then Compute $T(2,-3)$.

Question.7(15 points) (9.1)

a) Write the system $\begin{cases} y_1' = y_1 + 4y_2 \\ y_2' = 2y_1 + 3y_2 \end{cases}$ in terms of matrix notations.

b) Find the eigenvalues and the corresponding eigenvectors for the coefficient matrix A of the system.

c) Construct a matrix P that diagonalaizes A. Find P^{-1}.

d) Find the general solution and the particular solution that satisfies the initial conditions $y_1(0) = 0, y_2(0) = 0$.

Question.8 (12 points)(8.2) Let A be a matrix such that $A=\begin{bmatrix} 2 & 2 & -1 & 0 \\ -1 & -1 & 2 & -3 \\ 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

a) Solve the system $Ax=\begin{bmatrix}1 & 1 & 1 & -1\end{bmatrix}^T$.

b) Find the rank and nullity of A.

c) Determine a basis for the solution space of the system $Ax=\begin{bmatrix}1 & 1 & 1 & -1\end{bmatrix}^T$.

Question.9 (6 points)(7.1) Let $A=\begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & \frac{1}{2} & 3 & 8 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$. Find the eigenvalues of A.. Find the eigenvalues of A^7. Determine $Det(A)$.