Prince Sultan University

Department of Mathematical Sciences

MATH 223 – Final Examination

Term: 062

Monday, 4th June 2007

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Student Name: **Student ID #:**

Time allowed: 150 minutes **Maximum points: 100 points**

1. (6 points) Find the standard matrices for linear operators on R^2 represented by:
   a. A rotation of 60^0

   b. An orthogonal projection on the x-axis.

   c. The composotion of a rotation of 60^0, followed by an orthogonal projection on the x-axis, followed by a reflection about the line $y = x$.

2. (6 points) Find the volume of the parallelepiped determined by vectors $u = (2,-6,2)$, $v = (0,4,-2)$, and $w = (2,2,-4)$.

3. (8 points) Show that the points $(-1,-2,-3)$, $(-2,0,1)$, $(-4,-1,-1)$ and $(2,0,1)$ lie in the same plane.

4. (6 points) Find parametric equation for the line through $(-2,5,0)$ that is parallel to the planes $2x + y - 4z = 0$ and $-x + 2y + 3z + 1 = 0$.

5. (8 points) Determine wheather $p(x) = 1 - x + 2x^2$ and $q(x) = 2x + x^2$ are orthogonal on P_2 with respect to the inner product defined by $\langle p,q \rangle = \int_{-1}^{1} p(x)q(x)dx$.

6. (4 points) Let V be an inner product space. Show that if **u** and **v** are orthogonal vectors in V such that $\| u \| = \| v \| = 1$, then $\| u - v \| = \sqrt{2}$.

7. (6 points) Find the eigenvalues for eigenspace of A^{25} for $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$.

8. (10 points) Consider the basis $S = \{ v_1, v_2 \}$ for R^2, where $v_1 = (-2,1)$ and $v_2 = (1,3)$, and let $T : R^2 \longrightarrow R^3$ be the linear transformation such that $T(v_1) = (-1,2,0)$ and $T(v_2) = (0,-3,5)$.

Find a formula for $T(x_1, x_2)$, and use that formula to find $T(2, -3)$.

9. (8 points) Find $\ker(T)$ and determine whether the linear transformation $T : R^2 \longrightarrow R^3$, where $T(x, y) = (x - y, y - x, 2x - 2y)$ is one-to-one.

10. (8 points) Let $V = C[a,b]$ be the vector space of functions continuous on $[a,b]$, and let $T: V \longrightarrow V$ be the transformation defined by $T(f) = 5f(x) + 3\int_a^x f(t)\,dt$. Show that T is a linear operator.

11. (8 points) Let $T: R^2 \longrightarrow R^2$ be the linear operator defined by the formula

$$T(x, y) = (2x + y, x - 2y)$$

Determine whether T is one-to-one; if so find $T^{-1}(x, y)$

12. (12 points) Solve the system: $\begin{aligned} y_1' &= y_1 + 3y_2 \\ y_2' &= 4y_1 + 5y_2 \end{aligned}$ and find the solution that satisfies the initial conditions $y_1(0) = 2, y_2(0) = 1$.

13. (4 points) Calculate the distance between the point $(2, -5)$ and the line $y = -4x + 2$.

14. (6 points) Can we use the Wronskian to show that the set of vectors $\sin x, \cos x, x \sin x$ are linearly independent or not? (why?)