Prince Sultan University
Department of Mathematical Sciences

MATH 223 – Final Examination
12 June 2008

Time allowed: 180 minutes — Dr. Bahaa Eldin Abdalla
Maximum points: 100 points

1. (4 points) For which value(s) of the constant k does the system
$$x-y=3$$
$$2x-2y=-k$$
have no solution? Exactly one solution? Infinitely many solutions?

2. (5 points) Let $A^{-3}=\begin{bmatrix}-1 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & -1/8\end{bmatrix}$. Compute $tr(A)$ and $\det(A^T)$.

3. (4 points) Let $\begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix}=-3.$ Compute $\begin{vmatrix}-2a & -2b & -2c\\ -2g & -2h & -2i\\ -2d & -2e & -2f\end{vmatrix}$.

4. (5 points) Find the area of the triangle having vertices $P(1,-1,2)$, $Q(-1,3,4)$, $R(2,-1,1)$.

5. (6 points) Find four unit vectors that are ortogonal to $\mathbf{u}=(-1,2,2,-1)$.

6. (5 points) Find parametric equations for the line l passing through the points $P(2,4,-1)$ and $Q(5,-1,7)$. Where does the line intersect the *xy*-plane?

7. (4 points) Find an equation for the plane that passes through the origin and is parallel to the plane $7x+4y-2z+3=0$.

8. (5 points) Show that the linear operator $L:R^2\to R^2$ defined by $L(x,y)=(-x-3y,x-y)$ is one-to-one. Find the standard matrix for the inverse operator, and find $L^{-1}(w_1,w_2)$.

9. (4 points) Give an example to show that the set of all 2×2 matrices A such that $\det(A)=0$ is not a subspace of M_{22}.

10. (6 points) Consider the following homogeneous system

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(a) Show that the solution space of the system is a plane through the origin and find an equation for it.
(b) Determine a basis for and the dimension of the solution space of the system

11. (5 points) Show that $1+x+x^2$, $x+x^2$, and x^2 are three linearly independent vectors in P_2. Do they form a basis for P_2. Why?

12. (4 points) Let $\mathbf{v}_1 = (2,-3,1)$, $\mathbf{v}_2 = (4,1,1)$, and $\mathbf{v}_3 = (0,-7,1)$. Determine whether the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for R^3.

13. (5 points) Express $\mathbf{v} = (1,1)$ as a linear combination of $\mathbf{v}_1 = (1,-1)$, $\mathbf{v}_2 = (3,0)$, $\mathbf{v}_3 = (2,1)$ in three different ways.

14. (4 points) Sketch the unit circle in an *xy*-coordinate system in R^2 using the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{4}u_1v_1 + \frac{1}{9}u_2v_2$.

15. (6 points) Let $\mathbf{p} = x - 2x^2$ and $\mathbf{q} = 7 + 3x + 3x^2$ be two vectors in P_2. Define an inner product on P_2 and use it to find the angle between **p** and **q**.

16. (15 points) Find the eigenvalues and bases for the eigenspaces of

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$ Write a matrix P that diagonalizes A.

Compute A^{10} using the matrix P.

17. (4 points) Let V be an inner product space. Determine whether the function $T : V \to R$ defined by $T(\mathbf{u}) = \|u\|$ is a linear transformation. Justify your answer.

18. (9 points) Let $L : R^4 \to R^2$ be the function defined by $L(x, y, z, w) = (x + 2y + 3z, 4x + y + 5z + 2w)$.
(a) Find a basis for range (L) and a basis for Ker(L).
(b) Determine the rank and nullity of L.
