PRINCE SULTAN UNIVERSITY

MATH 111 **CALCULUS 1**

FINAL EXAM **14th JUNE 2011**

Start: **8:30 a.m.**
End: **10:30 a.m.**

Name: _____

I.D.: _____ **Instructor: Mr. Abid Bashir Zargar**

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 6 Question Sheets with 15 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the test.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2,3	14	
4,5,6	20	
7,8,9	18	
10, 11, 12	18	
13	12	
14,15	18	
TOTAL	100	

1) [6 points] Find the value of the following limits:

a) $y = \lim_{x \to 2} \dfrac{x^3 + 3x^2 - 12x + 4}{x^3 - 4x}$

b) $y = \lim_{t \to 9} \dfrac{t^2 - 81}{\sqrt{t} - 3}$

c) $y = \lim_{x \to \infty} \sqrt[3]{\dfrac{2 + 3x - 5x^2}{1 + 8x^2}}$

2) [4 points] Given that $y = \dfrac{\left(2\sqrt{x} + 1\right)\left(x - 1\right)}{x + 3}$ find $\dfrac{dy}{dx}$

3) [4 points] Given that $x^2 y + 3xy^3 - x = 3$ find $\dfrac{dy}{dx}$

4) [8 points] If $f(x)=2x^2+\sqrt{x}$ find $f'(x)$ using the formula $f'(x)=\lim_{x\to x_0}\dfrac{f(x)-f(x_0)}{x-x_0}$

5) [6 points] Show that if $y=\dfrac{\sin x}{1+\cos x}$ and $y=\dfrac{dy}{dx}$, then $x=\dfrac{\pi}{2}$.

6) [6 points] Find the *x* coordinate of the point(s), if any, at which the tangent line to the graph of $f(x)=\ln\left[\sqrt{6x-1}(4x+5)^3\right]$ is horizontal.

7) [6 points] Given $y = \dfrac{\cos^2 x \sin x \tan^3 x}{\sqrt{x}}$ find y' using logarithmic differentiation.

8) [4 points] Given that $y = \left(e^{5x} - 6^{\cos 3x}\right)^4$, find y'

9) [4 points] Given that $y = \pi^{x \tan x}$, find y'. **Simplify your answer as much as possible.**

10) [4 points] A spherical balloon is deflated so that its radius decreases at a constant rate of 15cm/min. At what rate must air be removed when the radius is 9cm?

11) [6 points] Given that $f(x)=(7x-3)(8+5x)$, find $f'''(x)$, the third derivative.

12) [6 points] Given that $f(x)=x^2\cos x+4\sin x$, find $f''(x)$, the second derivative.

13) [6 points] Given that $y=(\ln x)^{\ln x}$, find y'. **Simplify your answer as much as possible.**

14) [6 points] Show that for any constants *A* and *B*, the function $y=Ae^{2x}+Be^{-4x}$ satisfies the equation $y''+2y'-8y=0$

15) [6 points] Given that $y=\sqrt[3]{\tan^{-1}e^{x^2}}$, find y'

16) [12 points] Given that $f(x) = x^4 - 6x^2 + 5$, graph $f(x)$ by using the end behaviour, multiplicity, intercepts, first and second derivatives. Label the coordinates of the intercepts, stationary points and inflection points.

17) [12 points] The position function of a particle moving along a coordinate line is given by $s(t) = t^3 - 12t^2 + 36t - 20$ for $t \geq 0$, where *s* is in meters and *t* is in seconds.

a) At what time is the particle stopped?

b) Find the position, velocity, speed and acceleration at time $t = 1$

c) When is the particle speeding up? Slowing down?

d) Find the total distance travelled by the particle from time $t = 0$ to time $t = 5$.

18) [6 points] The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences along its other sides. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.