PRINCE SULTAN UNIVERSITY

MATH 111 CALCULUS I

FINAL EXAM 22nd JANUARY 2011

Start: 8:30 a.m.
End: 10:30 a.m.

NAME: I.D.

INSTRUCTOR: .

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 5 Question Sheets with 17 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the exam.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2,3	15	
4,5,6,7,8	24	
9,10,11,12	18	
13,14,15	21	
16,17	22	
TOTAL	100	
TOTAL	40	

1) [6 points] Find the value of the following limits:

a) $\lim_{x\to\infty}\sqrt[3]{\frac{6+4x-27x^2}{x^2-1}}$

b) $\lim_{x\to -4}\frac{2x+8}{x^2+x-12}$

2) [3 points] Discuss the continuity of $f(x)=\frac{1}{\sqrt{x}-1}$:

3) [6 points] Given that $y=2x^2-x$

a) Find the average rate of change of y with respect to x at over the interval $[1,2]$

b) Find the instantaneous rate of change of y with respect to x at an arbitrary value of x_0, using the limit of the Difference Quotient.

4) [6 points] Given that $f(x)=(7x-3)(8+5x)$, find $f''(x)$, the third derivative.

5) [6 points] Find all values of x at which the tangent line to the graph of $f(x)=\dfrac{x+3}{x+2}$ is perpendicular to the line $y=x$

6) [4 points] Given $y=\tan^{-1}(x^3)$, find $\dfrac{dy}{dx}$.

7) [4 points] Given that $x^3+y^3=3xy^2$, find $\dfrac{dy}{dx}$.

8) [4 points] Given that $y=e^{x\tan x}$, find $\dfrac{dy}{dx}$.

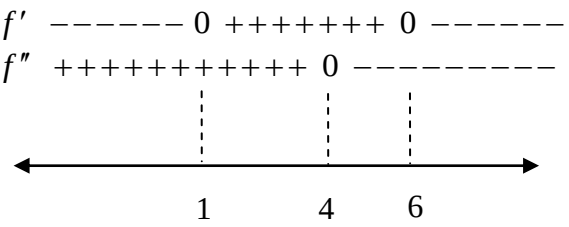
9) [4 points] Given $y = x^{\sin x}$, find $\frac{dy}{dx}$.

10) [4 points] Given that $y = \left[\sec^3\left(\ln\sqrt{x}\right)\right]$ find $\frac{dy}{dx}$.

11) [6 points] A 13ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2ft/s, how fast will the bottom be moving away from the wall when the top is 5ft above the ground?

12) [4 points] Given that $y = x^2 - 2x + 1$, find a formula for Δy.

13) [5 points] A sign graph f' and f'' of is given. Assuming f is continuous, find:



a) the intervals of increase

b) the intervals of decrease

c) intervals on which f is concave up

d) the intervals on which f is concave down

e) the x coordinates of all inflection points

14) [6 points] A rectangular plot of land is to be fenced using two kinds of fencing. Two opposite sides will use plastic fencing sold for \$2 per metre. The remaining two opposites sides will use metal fencing sold for \$4 per metre. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$8000?

15) [10 points] Given that $f(x) = x^4 - 6x^2 + 5$, analyze the signs of the first and second derivatives to find the local maximum and minimum of f, discuss concavity, find the points of inflection and sketch the graph.

16) [10 points] Graph the rational function $f(x) = \frac{8}{x^2 - 4}$.

17) [12 points] The position function of a particle moving along a coordinate line is given by $s(t) = 2t^3 - 6t^2$ for $t \geq 0$, where *s* is in meters and *t* is in seconds.

a) At what time is the particle stopped?

b) Find the position, velocity, speed and acceleration at time $t = 1$

c) When is the particle speeding up? Slowing down?

d) Find the total distance travelled by the particle from time $t = 0$ to time $t = 5$.