

PRINCE SULTAN UNIVERSITY**MATH 111****CALCULUS I****FINAL EXAM****JANUARY 30, 2010****Time Allowed: 150 minutes****Student Name:** _____**Student ID #:** _____**Section #:** _____**Teacher's Name:** _____

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 6 Question Sheets with 12 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the exam.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2	17	
3,4	21	
5,6,7	18	
8,9	16	
10	12	
11,12	16	
TOTAL	100	
TOTAL	40	

1) [12 points] Find the value of the following limits:

a) $\lim_{x \rightarrow -2} \frac{2x^2 + 5x + 2}{x^2 - x - 6}$

b) $\lim_{x \rightarrow \infty} \frac{4x^2 + 3x - 5}{1 - 2x^3}$

c) $\lim_{x \rightarrow -\infty} \frac{3x - 4}{\sqrt{9x^2 + 4}}$

d) $\lim_{x \rightarrow 0} \frac{\sin^2 2x}{2x^2}$

2) [5 points] Study the continuity of the function $f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 1 \\ \frac{x^2 - x}{x - 1} & \text{if } x > 1 \end{cases}$ at $x = 1$.

3) [5 points] Given that $f(x) = \sqrt{x}$. Use the definition of the derivative to find $f'(x)$.

4) [16 points] Find $f'(x)$

a) $f(x) = \sin 5x + (\sqrt{3x+1} - 2)^3$

b) $f(x) = e^{x^2} \tan^{-1} x$

c) $f(x) = \frac{2x^2 - x}{\ln(2x-1)}$

d) $f(x) = 2^x \sec^2 2x$

5) [6 points] Show that $y = x \ln x$ satisfies $x^2 y'' - xy' + y = 0$.

6) [6 points] Find the point, if any, on the graph of $f(x) = \frac{e^x - 1}{e^x + 1}$ at which the tangent line is horizontal.

7) [6 points] Find the equation of the tangent line to the curve $y^3 - xy = 1$ at $x = 0$.

8) [6 points] Use an appropriate local linear approximation to estimate the value of $\sqrt[3]{9}$.

9) [10 points] Find the relative maximum and minimum points for each of the following functions:

a) $f(x) = x + \cos 2x$, $x \in [0, \pi]$. (Use the second derivative test)

b) $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$. (Use the first derivative test)

- 10)** [12 points] Given that $f(x) = x^3 - 6x^2$, find the intervals on which f is increasing, decreasing, concave up, and concave down, and determine the relative maximum and minimum points and inflection points. Then sketch the graph of f . (**Determine the x and y-intercepts**)

11) [6 points] Find the dimensions of a rectangle with perimeter 20m whose area is as large as possible.

12) [10 points] The position function of a particle moving along a coordinate line is given by $s(t) = t^3 - 3t^2 - 9t$ for $t \geq 0$, where s is in meters(m) and t is in seconds(s).

a) At what time is the particle stopped?

b) When is the particle speeding up? Slowing down?

c) Find the total distance travelled by the particle from time $t = 0$ to time $t = 4$.