PRINCE SULTAN UNIVERSITY

MATH 111 CALCULUS I

FINAL EXAM 31st JANUARY 2009

Start: 1:00 p.m.
End: 3:30 p.m.

NAME: **I.D.**

INSTRUCTOR: .

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 7 Question Sheets with 11 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the exam.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1	**10**	
2,3	**10**	
4	**15**	
5,6,7	**15**	
8,9	**12**	
10	**10**	
11	**8**	
TOTAL	**80**	
TOTAL	**40**	

1) [10 points] Find the value of the following limits:

a) $\lim_{x\to\infty} \sqrt[3]{\frac{2+3x-x^2}{1+8x^2}}$

b) $\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3}$

c) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

d) $\lim_{x\to-2} \frac{3x^2+5x-2}{5x^2+13x+6}$

e) $\lim_{\theta\to 0} \frac{\theta^2}{1-\cos\theta}$

2) [4 points] Find the value of the constant *k* so that the function $f(x) = \begin{cases} kx^2 & \text{if } x \le 2 \\ 6x + k & \text{if } x \succ 2 \end{cases}$ is continuous everywhere.

3) [6 points] Given that $f(x) = x^3$

a) Find the average rate of change of f with respect to *x* over the interval $[1, 2]$

b) Find the instantaneous rate of change of f with respect to *x* at an arbitrary value x_0, using the Difference Quotient [$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$].

4) [15 points] Find $f'(x)$

a) $f(x) = \dfrac{\ln(3x+1)}{3x+1}$

b) $f(x) = e^{\sqrt{1+5x^3}}$

c) $f\ (x) = \sin x + 2\cos^3 x + \left(\dfrac{3x+1}{x^2}\right)^3$

d) $f(x) = e^x \sec^{-1} 2x$

e) $f(x) = x^{\tan x}$

5) [5 points] Given that $y = x\tan^{-1}x - \ln\sqrt{1+x^2}$, find $\dfrac{d^2y}{dx^2}$ in simplest form.

6) [5 points] Find the values of x at which the graph of the function $f(x) = (2x+7)^6(x-2)^5$ has horizontal tangent lines.

7) [5 points] Given that $\sin\left(x^2y^2\right) = x$, show that $\dfrac{dy}{dx} = \dfrac{1-2xy^2\cos\left(x^2y^2\right)}{2x^2y\cos\left(x^2y^2\right)}$

8) [4 points] The radius of a sphere is decreasing at a constant rate of 15cm/min. Find the rate of change of the volume when the radius is 9 cm? *Hint: The volume of a sphere is given by* $V = \frac{4}{3}\pi r^3$.

9) [8 points] Find the relative maximum and minimum points for each of the following functions:

a) $f(x) = \frac{1}{4}x^4 - x^3 + x^2$, using the second derivative test

b) $f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$, using the first derivative test

10) [10 points] Given that $f(x) = x^3 - 3x^2 - 24x$, find the intervals on which f is increasing, decreasing, concave up, and concave down, and determine the relative maximum and minimum points and inflection points. Then sketch the graph of f. (**Determine the *x* and *y*-intercepts**)

11) [8 points] The position function of a particle moving along a coordinate line is given by $s(t) = t^3 - 6t^2 + 1$ for $t \geq 0$, where *s* is in meters(m) and *t* is in seconds(s).

a) Find the position, velocity, speed and acceleration at time $t = 1$

b) At what time is the particle stopped?

c) When is the particle speeding up? Slowing down?

d) Find the total distance travelled by the particle from time t = 0 to time t = 5.