PRINCE SULTAN UNIVERSITY

MATH 111   CALCULUS I

FINAL EXAM   11th JUNE 2008

Start: 1:00 p.m.
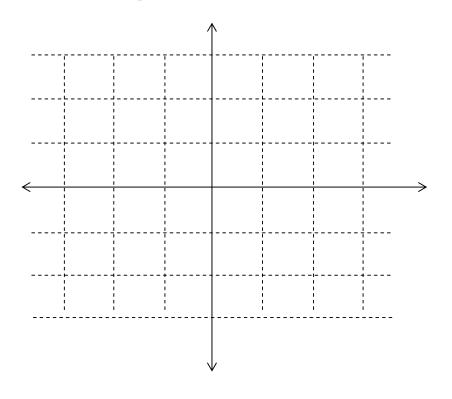**End: 3:15 p.m.**

Name:

I.D.

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 7 Question Sheets with 17 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the exam.
5. **Show all working out in the space provided.**

Question No.	Max. Points	Points Scored
1,2,3	**10**	
4	**10**	
5,6	**20**	
7,8,9	**16**	
10,11,12	**16**	
13,14,15	**14**	
16,17	**14**	
TOTAL	**100**	
TOTAL	**40**	

1) [4 points] Sketch the graph of $y = -2 + \sqrt{x+3}$. **Show clearly the location of the x and y intercepts.**

2) [3 points] Find the domain of $f(x) = \dfrac{\sqrt{2x-3}}{x^2 - 5x + 4}$

3) [3 points] Find an equation, in terms of m (the slope), for a family of lines that pass through the point $(1, -2)$.

4) [10 points] Given that $y = 2x^2$ and $x_0 = 0, \ x_1 = 1$.

a) Find the average rate of change of y with respect to x over the interval $[x_0, x_1]$.

b) Find the instantaneous rate of change of y with respect to x at the specified value of x_0.

c) Find the instantaneous rate of change of y with respect to x at a general unspecified value of x_0.

5) [8 points] Find the value of the following limits:

a) $\lim_{x \to \infty} \left(\sqrt{x^6 + 5x^3} - x^3 \right)$

b) $\lim_{y \to -\infty} \dfrac{2 - y}{\sqrt{7 + 6y^2}}$

6) [12 points] Find the value of the following limits:

a) $\lim_{x \to 2} \dfrac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$

b) $\lim_{k \to 4} \dfrac{k^2 - 16}{\sqrt{k} - 2}$

c) $\lim_{t \to -2} \dfrac{t^3 + 8}{t + 2}$

7) [8 points] If $f(x) = 2x^2 + 1$, find:

a) $f'(x)$ using the Difference Quotient $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

b) the equation of the tangent line to the graph $y = 2x^2 + 1$ at $x = -1$

8) [4 points] Find the value of $\lim\limits_{x \to 0} \dfrac{\sin^3 2kx}{x^3}$

9) [4 points] Given that $y = x^{-3}$, find $\left. \dfrac{d^4 y}{dx^4} \right|_{x=1}$

10) [4 points] Given that $y = \dfrac{4x+1}{x^2-5}$, find $\left.\dfrac{dy}{dx}\right|_{x=1}$

11) [6 points] Given that $y = x^2 \cos x + 4\sin x$, find $\dfrac{d^2y}{dx^2}$

12) [6 points] Given that $y = \ln\left[\dfrac{\cos x}{\sqrt{4-3x^2}}\right]$, find $\dfrac{dy}{dx}$ **in its simplest form.**

13) [4 points] Given that $y = \ln\left(1 - xe^{-x}\right)$, show that $\dfrac{dy}{dx} = \dfrac{x-1}{e^x - x}$

14) [4 points] Given that $y = 3\cot^4 x$, find the equation of the tangent line at $x = \dfrac{\pi}{4}$ in point-slope form and slope–intercept form.

15) [6 points] Given that $xy + y^2 = 2$,

    a) Show that $\dfrac{dy}{dx} = \dfrac{-y}{x + 2y}$

    b) Show that $\dfrac{d^2 y}{dx^2} = \dfrac{2y\left(x + y\right)}{\left(x + 2y\right)^3}$

16) [8 points] Given that $f(x) = x^4 - 6x^2 + 5$, analyze the signs of the first and second derivatives to find the local maximum and minimum of f, discuss concavity, find the points of inflection and sketch the graph.

17) [6 points] An open box is to be made from a 3-ft by 8-ft rectangular piece of metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume that the box can have.