PRINCE SULTAN UNIVERSITY

MATH 223 – Linear Algebra

Final Examination

Semester I, Term 091

Saturday, January 30, 2010

Time Allowed: 150 minutes

Name: ______________________________________

I.D. ______________________________

Instructors Name: ____________________ **Section:** ________

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 5 Question Sheets with 7 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the test.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2	32	
3,4	21	
5	20	
6	16	
7	11	
TOTAL SCORE	100	

Q.1: *Write True(T) or False(F) for each of the following statements.* (24pts)

1) The angle between the vectors $\mathbf{u} = (2, 2, 0)$ and $\mathbf{v} = (-1, 0, 1)$ is 60°. (------)

2) Let $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{v} = \overrightarrow{PQ}$ where P is $(1,-1)$ and Q is $(-5,7)$. Then $\|\mathbf{v}\| = 2\|\mathbf{u}\|$ and $\mathbf{v}$ is in the opposite direction of $\mathbf{u}$. (------)

3) The distance between the point (2, 1) and the line $3x + 4y = 1$ is 2. (------)

4) The set R^2, with the standard addition operation and the following scalar multiplication $k(x, y) = (kx, 0)$, is a vector space. (------)

5) The set $W = \{A \in M_{22} \mid A \text{ is symmetric}\}$ is a subspace of M_{22} and $\dim(W) = 3$. (------)

6) The set $S = \{(1,0,2,0), (0,1,0,0), (0,1,1,1)\}$ is a linearly independent set in R^4. (------)

7) Consider the vector space $C[1,4]$, with the inner product $\langle f, g\rangle = \int_1^4 f(x)g(x)dx$. (------)
If $f(x) = x$ and $g(x) = 1$, then $d(\mathbf{f}, \mathbf{g}) = 3$.

8) If $\lambda = 0$, $\lambda = -1$ are the eigenvalues of a 2×2 matrix A with $\{(1,1)\}_{\lambda=0}, \{(1,2)\}_{\lambda=-1}$ as bases for the corresponding eigenspaces, then $A^{10} = \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$. (------)

9) If A is any $n \times m$ matrix and $B = AA^T$, then B is always diagonalizable. (------)

10) The function $T: M_{nn} \to R$, defined by $T(A) = \det(A)$, is a linear transformation. (------)

11) The standard matrix of the composition of a reflection about the x-axis followed by an orthogonal projection on the y-axis is $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$. (------)

12) If $T: V \to W$ is an onto linear transformation and $\dim(V) = 6, \dim(W) = 3$, then $nullity(T) = 2$. (------)

Q.2: Find an equation of the plane that contains the line $x = t, y = 1 + 3t, z = 2 - t$ and is perpendicular to the plane $x - y + 2z = 5$ (8 pts)

Q.3: Let $\mathbf{u} = (4, -1, 2)$ and $\mathbf{v} = (1, 0, 3)$. Find (10pts)

(a) The vector component of $\mathbf{u}$ orthogonal to $\mathbf{v}$.

(b) If $\mathbf{w} = (k, 2, -1)$, find the constant k such that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ lie in the same plane.

Q.4: Consider the linear operator $T : R^3 \to R^3$ defined by $\begin{cases} w_1 = x_1 + 5x_2 + 2x_3 \\ w_2 = x_1 + 2x_2 + x_3 \\ w_3 = -x_1 + x_2 \end{cases}$. (11pts)

Find bases for the kernel and the range of *T*, and determine whether *T* is one-to-one.

Q.5: Consider the set $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = (1,0,-1), \mathbf{u}_2 = (-1,1,2), \mathbf{u}_3 = (1,2,0)$. (20pts)

(a) Show that S is a basis for R^3.

(b) If $\mathbf{v} = (x_1, x_2, x_3)$, find in terms of x_1, x_2, x_3 the coordinate vector of $\mathbf{v}$ relative to the basis S.

(c) If $T : R^3 \to R^3$ is a linear operator such that $T(\mathbf{u}_1) = (1,-1,0), T(\mathbf{u}_2) = (-1,2,1), T(\mathbf{u}_3) = (1,-1,1)$, use the result of part (b) to deduce that T has the formula $T(x_1, x_2, x_3) = (x_1, x_1 - x_2 + 2x_3, x_1 + x_3)$.

(d) Show that T is one-to-one, and then find the formula of T^{-1}.

Q.6: (a) Find the eigenvalues and bases for the corresponding eigenspaces of the matrix $A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$. (16pts)

(b) 1- Find the general solution of the system $\begin{cases} y_1' = 2y_2 \\ y_2' = -y_1 + 3y_2 \end{cases}$

2- Find the particular solution that satisfies the initial conditions $y_1(0) = 3, y_2(0) = 1$.

Q.7: (a) Show that $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 - u_1 v_2 - u_2 v_1 + 2u_2 v_2$ is an inner product on R^2. (11pts)

(b) Show that the vectors $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (1, 2)$ are orthogonal in this inner product space.