PRINCE SULTAN UNIVERSITY

MATH 221 – Numerical Analysis

Final Examination

Semester 2, Term 092

Saturday, June 12, 2010

Time Allowed: 150 minutes

Name: _____

I.D. _____

Instructors Name: _____ **Section:** _____

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 5 Question Sheets with 5 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the test.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1	8	
2	8	
3	8	
4	8	
5	8	
TOTAL SCORE	40	

Q1. Consider the equation $x^3 + 10x - 1 = 0$.

a) Use the Bisection method for this equation on the interval [0, 0.1] to find p_2.

b) Use Newton's method, with $p_0 = 0$, to approximate the solution of this equation accurate to within 10^{-2}.

Q2. Given the following data

x	8.1	8.3	8.5	8.7
$f(x)$	16.9441	17.56492	18.19056	18.82091

a) Use appropriate Lagrange interpolating polynomial of degree two to approximate $f(8.6)$.

b) Use the most accurate three-point formula to approximate $f'(8.5)$.

Q3. a) Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = e^{x^2}$ about $x_0 = 0$. Then, approximate $\int_0^{0.5} f(x)dx$ using $\int_0^{0.5} P_3(x)dx$.

b) Approximate the integral $\int_0^{0.5} e^{x^2} dx$ using the Simpson's rule.

Q4. Given the initial value problem: $y' = 1 + \frac{1}{t}y, \quad 1 \le t \le 1.5 \qquad y(1) = 2$

a) Use Euler's method and the Midpoint method, with $h = 0.25$, to approximate the solution.

b) Find the exact solution and then the actual errors in the Midpoint method.

t_i	**(method 1)** Euler's method w_i	**(method 2)** Midpoint method w_i	**(method 2)** Actual error $\lvert y(t_i) - w_i \rvert$

Q5. Given the linear system $\begin{cases} 10x_1 - x_2 = 9 \\ x_1 - 10x_2 + 2x_3 = -7 \\ -2x_2 + 10x_3 = 8 \end{cases}$

a) Use Gaussian elimination to find the exact solution of this system.

b) Approximate the solution of the above system using Gauss-Seidel iterative method, with tolerance 10^{-3} and initial approximation (0, 0, 0).

A list of formulas

The three-point formulas

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)] + \frac{h^2}{3} f'''(\xi)$$

$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6} f'''(\xi)$$

Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}[f(a) + 4f(x_0) + f(b)] - \frac{h^5}{90} f^{(4)}(\xi)$$

Euler's method

$$w_{i+1} = w_i + hf(t_i, w_i)$$

Midpoint method

$$w_{i+1} = w_i + hf(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i))$$