PRINCE SULTAN UNIVERSITY
MATH 221
Final Examination
Semester I, Term 081
Monday, February 9, 2009

Time Allowed: 120 minutes

Student Name: **Student ID #:**

Teacher's Name: **Section #:**

Important Instructions:

1. **You may use a scientific calculator that does not have programming or graphing capabilities.**
2. **You may NOT borrow a calculator from anyone.**
3. **You may NOT use notes or any textbook.**
4. **There should be NO talking during the examination.**
5. **Your exam will be taken immediately if your mobile phone is heard.**
6. **Looking around or making an attempt to cheat will result in your exam being cancelled.This examination has 6 problems, some with several parts. Make sure your paper has all these problems.**

Problems	Max points	Student's Points
Q.1	10	
Q.2	9	
Q.3	8	
Q.4	8	
Q.5	9	
Q.6	6	
Total	50	

<u>Question.1</u> (10 pts.)

Answer by writing True (T) of False (F) for each of the following statements:

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1. Bisection's method provides only one root in a certain interval. (----------)
2. Newton's method has convergence of order 3. (----------)
3. The need to know the value of the derivative of f in Newton's method is a major weakness. (----------)
4. Secant's method starts with one initial approximations. (----------)
5. If p^* is an approximation to p, the relative error is $|p - p^*|$ and the absolute error is $\dfrac{|p - p^*|}{|p|}$ provided that $p \neq 0$. (----------)
6. Richardson's extrapolation is used to generate low-accuracy results while using low order formulas. (----------)
7. Simpson's rule results from integrating over $[a,b]$ the second Lagrange polynomial with nodes $x_0 = a,\ \ x_1 = a + h$ and $x_2 = b$ where $h = \dfrac{b-a}{2}$. (----------)
8. The objective of Euler's method is said to obtain an approximation to the well--posed initial value problem $y'' = f(t, y),\ \ a \le t \le b,\ \ y(a) = c.$ (----------)
9. If a continuous function f is prescribed on an interval $[a,b]$ and if interpolating polynomials p_n of higher degree are constructed for f, the natural expectation is that these polynomials will converge to f uniformly on $[a,b]$. (----------)
10. An algorithm for computing a divided difference table can be very efficient and is recommended as the best means for producing an interpolating polynomial. (----------)

<u>Question.2</u> (9 pts.)

Consider the initial value problem (IVP) $\begin{cases} y' = te^{3t} - 2y, 0 \le t \le 1 \\ y(0) = 0 \end{cases}$ with $h = 0.5$.

a) Use Taylor's method of order 2 to approximate the solutions of the (IVP).

b) Use Taylor's method of order 4 to approximate the solutions of the (IVP).

c) Construct a table to compare results obtained in part (a) and (b) and the exact solution of (IVP): $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$.

<u>Question.3</u> (8 pts.) Evaluate the integrals

a) $\int_1^3 x^2 \ln x dx$ using composite Trapezoidal rule with $n = 4$.

b) $\int_1^3 x^2 \ln x dx$ using composite Simpson's rule with $n = 4$.

Question.4 **(8 pts.)** Solve the system $\begin{cases} 4x+2y+z=11 \\ -x+2y=3 \\ 2x+y+4z=16 \end{cases}$ with $X^{(0)}=[1 \quad 1 \quad 1]^T$.

Obtain $X^{(1)}$ and $X^{(2)}$.

a) Using Jacobi method with initial vector

b) Using Gauss Seidel method with initial vector $X^{(0)}=[1 \quad 1 \quad 1]^T$. Obtain $X^{(1)}$ and $X^{(2)}$.

<u>Question.5</u> (9 pts.)

a) Find a polynomial $f(x)$ using Lagrange interpolation where $f(0)=1, f(1)=2$ *and* $f(2)=7$.

b) Write the Newton interpolating polynomial for the data (Use divided difference algorithms):

x	4	2	0	3
$f(x)$	63	11	7	28

c) Use Newton's method to obtain an iteration formula for $\sqrt{17}$.

<u>Question.6</u> (6 pts.)

a) Derive a numerical differential formula of $o(h^4)$ by applying Richardson's extrapolation to

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{6} f'''(x) - \frac{h^4}{120} f^{(5)}(x).$$

b) For the polynomial $p(z) = 3z^5 - 7z^4 - 5z^3 + z^2 - 8z + 2.$ Find a disk centered at the origin that contains all the zeros.

Formulas:

1. $$\int_a^b f(x)dx = \frac{1}{2}h\left[f(a) + 2\sum_{i=1}^{n-1} f(a+ih) + f(b) \right].$$

2. $$\int_a^b f(x)dx = \frac{1}{3}h\left[f(x_0) + 2\sum_{i=2}^{n/2} f(x_{2i-2}) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + f(x_n) \right].$$

3. $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

4. $$\rho = 1 + |a_n|^{-1} \cdot \max_{0 \le k \le n} |a_k|.$$