Prince Sultan University
Department of Mathematics and Physical Sciences

Math 111
Final Examination

Semester II, Term 092
Tuesday, June 22, 2010
Time Allowed: 120 minutes

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|---|
| Student Name: |
| Student ID #: |
| Teacher's Name: |

***Important Instructions:***

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 10 problems, some with several parts. Make sure your paper has all these problems.

| Problems | Max points | Student's Points |
|---|---|---|
| 1 | 20 | |
| 2,3 | 15 | |
| 4,5,6 | 16 | |
| 7,8,9 | 17 | |
| 10 | 12 | |
| | | |
| Total | 80 | |

Q.1 (20 pts, 2 points each) Answer the following quick questions by **writing only the final result**:

| | Questions | Answers |
|---|---|---|
| 1. | For $f(x) = -4\sin(\frac{x}{3} + 2\pi)$. Find amplitude and period. | |
| 2. | Let $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$. Evaluate $(f \circ g)(2) + (f + g)(1)$. | |
| 3. | Evaluate $\lim_{x \to -\infty} (1 + 2x^3 - 5x^5)$ | |
| 4. | Find $y'''$ for $y = x^{-5} + x^5$. | |
| 5. | Find the relative extrema for $f(x) = 2x^3 - 24x$. | |
| 6. | Given $f(x) = \begin{cases} x - 2, & x < 0 \\ x^2, & 0 \le x \le 2 \\ 2x, & x > 2 \end{cases}$. Find $\lim_{x \to 2^+} f(x)$. | |
| 7. | Evaluate $\lim_{x \to 0} \frac{\sin 6x}{\sin 14x}$. | |
| 8. | The average rate of change of $f(x) = x^2$ with respect to $x$ over $[1,3]$ is | |
| 9. | Find $\frac{dy}{dx}$ for $y = \cos^{-1} x^2$. | |
| 10. | Find the inflection points for $f(x) = x^4 - 3x^3$. | |

Q.2 (9 points) Evaluate the limits

a) $\lim_{x\to -1} \dfrac{2x^2 + x - 1}{x + 1}$.

b) $\lim_{y\to 4} \dfrac{4 - y}{2 - \sqrt{y}}$.

c) $\lim_{t\to \infty} \dfrac{6 - t^3}{7t^3 + 3}$.

Q.3 (6 points) Find the values of $a$ and $b$ that make $f(x) = \begin{cases} \dfrac{2\sin x}{x}, & x < 0 \\ a, & x = 0 \\ b\cos x, & x > 0 \end{cases}$ continuous.

Q.4 (6 points) Find the derivatives of

a) $y = e^{x^3} \sec^{-1} x.$

b) $y = \sqrt{3x^2 + 6x - 9} + \sec x \tan x.$

Q.5 (5 points) Find all values of $x$ at which the tangent line to the curve $y = \dfrac{x^2 - 1}{x + 2}$ is horizontal.

Q.6 (5 points) Use the logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = (x^3 - 2x)^{\ln x}$.

Q.7 (6 points) Given $f(1) = -2$ and $f'(1) = 3$.
Find the equation of the tangent line to $h(x) = (x^2 + 2x) f(x)$ at $x = 1$.

Q.8 (6 points) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 80 running feet of chicken wire is available for the fence?

Q.9 (5 points) Find $\frac{dy}{dx}$ by implicit differentiation of $\tan(xy^2 + y) = x$.

Q.10 (12 points) Consider the rational function $f(x) = \dfrac{x^2 - 1}{x^3}$.

a) Check symmetry of $f(x)$.

b) Find $x$- and $y$- intercepts.

c) Write equations of vertical and horizontal asymptotes.

d) Find the critical points, local maximum and minimum and interval of increase and decrease.

e) Find interval of concave up and concave down.

f) Sketch the graph.