Prince Sultan University
Department o f Mathematical Sciences
Final Exam
FALL, 2009   (091)
January 30, 2010
MATH 113 – CALCULUS II

Time Allowed   : 150 minutes ($2\frac{1}{2} hours$ **)**

Maximum Points: 100 points

Name of the student: _____

ID number   : _____

Section   :   **221**

For All The Students:

- **Answer all the questions.**

- **This exam consists of _a total of 7 pages and 9 questions._**

- **Show your working in the space provided for each question.**

- **Show all the key steps of your work.**

- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	Your Score
1,2,3	16	
4,5	20	
6	12	
7	40	
8,9	12	
Total	**100**	

40

Q.1 (8 points): Evaluate the given limit:

1) $\lim\limits_{x\to 0} \dfrac{x-\sin x}{x^3}$

2) $\lim\limits_{x\to\infty} x\sin\dfrac{1}{x}$

Q.2 (4 points): Find all values of x^* that satisfy the Mean Value Theorem for the following function over the given interval.

$$f(x)=\sqrt{x} \qquad ; \quad [0,3]$$

Q.3 (4 points): Evaluate the following: $\lim\limits_{n\to+\infty}\sum\limits_{k=1}^{n}\dfrac{3k}{n^2}$

Q.4 (10 points): Let R be the region bounded by the graphs of $y = 4x$ *and* $y = x^2$

      (a) Find the area of R.

      (b) Find the volume of the solid generated when R is rotated about the $y - axis$.

Q.5 (10 points): Evaluate the improper integral and determine whether it converges or diverges

      (i) $\displaystyle \int_{0}^{2} \frac{1}{(x-1)^{\frac{2}{3}}} dx$

      (ii) $\displaystyle \int_{e}^{+\infty} \frac{1}{x \ln^3 x} dx$

Q.6 (12 points): Use $n = 6$ to approximate the integral $\displaystyle\int_{0}^{3} \sqrt{x^3 + 1}\, dx$

    (i) using the Trapezoid Method

    (ii) using the Simpson's Rule.

    (iii) Use your calculator to compute exact value of the integral rounded to 4 decimal places.

    (iv) Find the absolute error of both approximations.

Q.7 (40 points): Evaluate the following integrals:

a) $\displaystyle\int \frac{\sec^2 x \tan x}{\sqrt{9 - \tan^2 x}} dx$

b) $\displaystyle\int \frac{e^x}{4 + e^{2x}} dx$

b) $\displaystyle\int \frac{2x^2 + 3}{x(x-1)^2} dx$

c) $\int \cos x \ln(\sin x)dx$

c) $\int \dfrac{3x^3}{\sqrt{4-x^2}}dx$

e) $\int \sec^4 x\, dx$

f) $\int_{0}^{\frac{1}{2}} \tan^{-1}(2x)\,dx$

Q.8 (6 points): Solve the initial value problem: $\dfrac{dy}{dx} + 4y = -3e^{-2x}$; $y(0) = 2$

Q.9 (6 points): Solve the differential equation: $y' = \dfrac{2}{xy + y}$