MATH 111 – CALCULUS I

Final Exam 082

Monday 29th June, 2009

150 minutes

Name : _____

ID number : _____

Section : _____

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 7 pages and 12 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	Max. Points	Points Scored
Q.1	4	
Q.2	6	
Q.3	6	
Q.4	6	
Q.5	20	
Q.6	21	
Q.7	5	
Q.8	10	
Q.9	4	
Q.10	6	
Q.11	6	
Q.12	6	
Total	100	
TOTAL	40	

Q.1: Find the natural domain of $f(x)=\dfrac{x+1}{\sqrt{x^2-4}}$

Q.2: Find the values of a and b so that the following function is continuous at $x=2$

$$f(x)=\begin{cases} x^2+ax+2 & x<2 \\ 5+b & x=2 \\ bx+4 & x>2 \end{cases}$$

Q.3: Use the definition of the derivative to find $f'(x)$ if $f(x)=x^2-x$.

Q.4: Find the equation of the tangent line to the graph of $f(x)=x^3-2x^2+4x+2$ at $x=1$

Q.5: Find the limits:

a) $\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{3x+2}$

b) $\lim_{x\to 1} \frac{x^3-3x^2+2x}{x-1}$

c) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2}$

d) $\lim_{x\to 0} \frac{3x-\sin(kx)}{x}$, $k \neq 0$

e) $\lim_{x\to-\infty} \frac{3}{x+4}$

Q6: Find the derivative, $\frac{dy}{dx}$ (simplify your answer):

a) $y = e^{3x}.(\cos 2x)^3$

b) $x^2e^y + y^2e^x = 2e$

c) $xe^{(x^2+y^2)} = 5$

d) $y = \sqrt{\cos^{-1} x^2}$

Q6 continued: Find the derivative, $\frac{dy}{dx}$ (simplify your answer):

e) $y = \sqrt[3]{\frac{(x^2+5)\tan^4 2x}{(x^3-8)}}$

f) $y = \ln\left(\frac{\sqrt{x}.\sqrt[3]{x+1}}{\sin x.\sec x}\right)$

g) $y = (3x^2+5)^{\frac{1}{x}}$

Q.7: A stone dropped into a still water sends out a circular ripple whose radius increases at a constant rate of $4m/s$. Find the rate at which the area enclosed by the ripple is increasing at the end of $10\,\text{sec}onds$.

Q.8: Let $f(x) = (x^2 - 1)^3$

a) Find all the critical numbers.

b) Find the intervals where $f(x)$ is increasing or decreasing.

c) Determine where the relative maxima and relative minima occur, if any.

d) Determine the open intervals on which $f(x)$ is concave up and concave down and give the inflection points.

e) Sketch the graph of $f(x)$

Q.9: Let $f(x) = \frac{1}{x-2}$. Find the average rate of change of y with respect to x over the interval $[3,5]$

Q10: Let $f(x) = \begin{cases} |x+1| & x \le -2 \\ x+1 & -2 \le x < 1 \\ \sqrt{x+3} & 1 \le x \le 6 \\ \dfrac{6}{8-x} & x > 6 \end{cases}$

Find the values of x, if any, at which f is not continuous.

Q.11: A container with square base, vertical sides, and open top is to be made from 1200 ft^2 of material. Find the dimensions of the container with the greatest volume.

Q.12: Let $S(t) = 2t^3 - 3t^2 - 12t + 10$ be a position function of an object moving along a line, where S is meters and t is in seconds.

a) At what time is the object stopped?

b) Determine the intervals on which the object is speeding and the intervals on which it is slowing down.

c) Find the total distance traveled by the particle from time t = 0 to time t = 5.