Prince Sultan University

Department of Mathematical Sciences

Semester I, 2011 FALL (Term 111)
January 18, 2012

MATH 111 – Calculus I
Final Exam

Time Allowed : 120 minutes
Maximum Points : 100 points

Name of the student: _____

ID number : _____

Section : _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 9 problems, some with several parts and a total of 6 pages. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1	20	
Q.2 , Q.3 , Q.4	19	
Q.5	25	
Q.6 , Q7 , Q.8	18	
Q.9	18	
Total	100	

Show Your Steps

Q.1 (20 points): Find the derivative, $\frac{dy}{dx}$. **(Simplify as much as possible)**

(i) $y = x^{\cos 4x}$

(ii) $y = x^2 + \frac{x}{\sqrt{3x - x^2}}$

(iii) $x^3 y - 3y \sin x = xe^{3y}$

(iv) $y = x^3 \cos(\ln x^2)$

(v) $y = \sin\left(\tan\sqrt{1+x^2}\right)$

Q.2 (8 points): Show that $f(x)=\begin{cases} x^2+2 & \text{if } x \le 1 \\ x+2 & \text{if } x>1 \end{cases}$ is continuous but not differentiable at $x=1$.

Q.3 (5 points): Use the **Second Derivative Test** to determine all local extrema, if any for $f(x)=1+10x^2-5x^3$

Q.4 (6 points): Given that $f(x)=\dfrac{5}{(x^2+4)^5}$. Find $\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$.

Q.5 (25 points): Evaluate the limits:

(i) $\lim_{x \to -1} \dfrac{x^2+6x+5}{x^2-3x-4}$

(ii) $\lim_{x \to -\infty} \dfrac{\sqrt{3x^2-2}}{x+5}$

(iii) $\lim_{x \to 1} \dfrac{x-1}{\sqrt{10-x}-3}$

(iv) $\lim_{x \to 0} \dfrac{x^2-\tan^{-1}x}{x\cos x}$

(v) $\lim_{x \to 0} \dfrac{\tan 2x}{x\cos x}$ without using L'Hopital's Rule

Q.6 (6 points): For the function $f(x) = x + \ln x$, check the hypotheses of Mean Value Theorem on the interval $[1, e]$ and find a value of c that makes the appropriate conclusion true.

Q.7 (6 points): Find the point on the curve $y = \sqrt{x+1}$ closest to the point $(1,1)$.

Q.8 (6 points): Two ships started sailing from the same location at noon. Ship A travels north at 70km/h, while ship B travels east at 30km/h. After three hours, what is the rate of change of the distance between the two ships?

Q.9 (18 points): Consider the function $f(x)=\dfrac{3x^2}{x^2+1}$.

(a) Find the domain and intercepts of f .

(b) Find the horizontal and vertical asymptotes of f if they exist.

(c) Find critical number(s) for f and the intervals where f is increasing or decreasing.

(d) Find the local extrema of f , if any.

(e) Find the inflection point(s) of f and the intervals where f is concave up or down.

(f) Sketch the graph of f showing all significant features.