1) Consider the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

   a) Find a matrix P that **orthogonally diagonalizes** A**.**

   b) Use the first part to calculate A^{11}.

2) Consider the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$.

a) Given that $\mu = 1$ is an eigenvalue for A, find the characteristic polynomial and all the eigenvalues of A.

b) Find a basis and the dimension for the kernel of the linear operator $T: R^3 \to R^3$ whose standard matrix representation is $[T] = A - I_3$. Is the matrix A diagonalizable ? Why?

c) Find the eigenvalues of A^{13} and $B = \begin{bmatrix} 2 & -2 & -2 \\ 1 & 5 & 1 \\ -1 & -1 & 3 \end{bmatrix}$.

3) Evaluate the triple integrals:

a) $\int_{0}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} yz\,dz\,dy\,dx.$

b) $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\,dz\,dy\,dx.$

4) Consider the set $S = \{p_1 ,\ p_2 ,\ p_3\},\ p_1 = x, p_2 = \frac{-4}{5} + \frac{3}{5}x^2, p_3 = \frac{3}{5} + \frac{4}{5}x^2$ of vectors in the space P_2 of all polynomials with degree less than or equal 2.

   a) Verify that S is an orthonormal basis P_2.
   b) Find $(p)_S$, the coordinate vector corresponds to the vector p with respect to the basis S, given that $p = 1 + 5x + 5x^2$.
   c) Find the polynomial $q \in P_2$, given that $(q)_S = (1,1,1)$.

5) a) Find the distance between the point $P(1,1,1)$ and the plane passing through the points $P_1(2,0,3), P_2(-1,1,3)$ and $P_3(0,1,2)$.

b) Find parametric equations for the line of intersection of the planes :

$x - 2y + 3z = -1$ and $-3x + y + 2z + 4 = 0$.

c) Find an equation for the plane passing through $P(-1,1,0)$ that is perpendicular to the line

$$x - 1 = 2t, y - 2 = 3t, z = -5t, -\infty < t < \infty.$$