

Prince Sultan University

Department of Mathematical Sciences

Semester I, 2012 FALL (Term 121) January 14, 2013

MATH 111 – Calculus I Final Exam

Time Allowed : 120 minutes Maximum Points : 100 points

Name of the student: _____

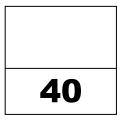
ID number

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Section 250	Section 224	Section 249	Section 223
10 11	11 12	8 9	10 11

Important Instructions:

- 1. You may use a scientific calculator that does not have programming or graphing capabilities.
- 2. You may NOT borrow a calculator from anyone.
- 3. You may NOT use notes or any textbook.
- 4. There should be NO talking during the examination.
- 5. Your exam will be taken immediately if your mobile phone is seen or heard
- 6. Looking around or making an attempt to cheat will result in your exam being cancelled
- 7. This examination <u>has 14 problems</u>, some with several parts and a total of 7 pages. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1 , Q.2	22	
Q.3, Q.4, Q.5, Q.6	18	
Q.7 , Q.8 , Q.9	15	
Q.10	16	
Q.11 , Q.12	16	
Q.13 , Q.14	13	
Total	100	



Q.1 (16 points): Find the derivative, $\frac{dy}{dx}$. (Simplify as much as possible) (i) $y = 2^{\sqrt{3+\cos(x)}}$

(ii)
$$y = e^{x^2} \sin(2x)$$

(iii)
$$y = \cot^3 \left(\cos \left(\frac{x}{3} \right) \right)$$

(iv)
$$y = \log(x) + \log_x(10) + \log_x(x) + \log(10)$$

O.2 (6 points): Let
$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0\\ 0 & \text{if } x = 0 \end{cases}$$

Show that f is differentiable at x = 0.

Q.3 (4 points): Let $f(x) = \cos(2x-3)$. Find $f^{(2013)}(x)$, the 2013th derivative of f.

Q.4 (4 points: If
$$g(x) = (x^2 + 2x + 3)f(x)$$
, $f(0) = 5$, and $\lim_{x \to 0} \frac{f(x) - 5}{x} = 4$.
Find $g'(0)$

Q.5 (5 points): Find the points on the curve: $y = x^3 - 11x + 5$ at which the tangent line to the curve has the equation: y + 11 = x.

<u>Q.6 (5 points)</u>: Let $f(x) = x^3 - 6x^2 + ax + b$ If Rolle's Theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$ on [1,3], find *a* and *b*. **<u>Q.7 (5 points)</u>**: Let $f(x) = \begin{cases} \frac{1 - \cos(x)}{x^3 - x^2} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$. Find k so that f is continuous at x = 0

<u>Q.8 (5 points)</u>: Let $f(x) = \frac{x-1}{x+1}$

- Find the vertical and horizontal asymptotes of f, if any. (a)
- Find $(f \circ f)(x)$ and its domain. (b)

<u>Q.9 (5 points)</u>: Find the point(s) P(x, y) on the curve $y = x^2$ that is(are) closest to the point(0,1).

Q.10 (16 points): Evaluate the following limits:

(i)
$$\lim_{x \to 4} \frac{x^3 - 7x^2 + 12x}{4 - x}$$

(ii)
$$\lim_{x \to 0} \frac{\tan(7x) + 2x}{3x}$$

(iii)
$$\lim_{x \to -\infty} \left(\sqrt{2x^2 - x} + 3x \right)$$

(iv)
$$\lim_{x \to 3} \frac{-2}{(x-3)^3}$$

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<u>Q.11 (8 points)</u>: Let $f(x) = xe^{x-x^2}$.

- (a) Find the critical number(s) for f
- (b) Find the intervals on which f is increasing and/or decreasing.
- (c) Find the local maximum and/or local minimum, if any.

Q.12 (8 points): (i) Show that if $y = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$, where a > 0, then $y' = \sqrt{a^2 - x^2}$

(ii) Show that if
$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$
, then $y' = \frac{2}{1+x^2}$

<u>Q.13 (10 points)</u>: Let $f(x) = \frac{x^2}{x^2 + 1}$.

- (a) Find the intervals on which f is concave up or concave down.
- (b) Find the inflection point(s) of f, if any.
- (c) Sketch the graph of f showing all significant features.

<u>Q.14 (3 points)</u>: Let $f(x) = e^{\frac{x}{2}}g(e^{-x})$ be a positive function. Given that g is differentiable and concave up, Show that f is concave up.