Prince Sultan University

Department of Mathematical Sciences

Semester I, 2012 FALL (Term 121)
January 14, 2013

MATH 111 – Calculus I
Final Exam

Time Allowed : 120 minutes
Maximum Points : 100 points

Name of the student: _____

ID number : _____

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Section 250	Section 224	Section 249	Section 223
10 ---- 11	11 ----- 12	8 ----- 9	10 ----- 11

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination **has 14 problems** , some with several parts and a total of 7 pages. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1 , Q.2	22	
Q.3 , Q.4 , Q.5 , Q.6	18	
Q.7 , Q.8 , Q.9	15	
Q.10	16	
Q.11 , Q.12	16	
Q.13 , Q.14	13	
Total	100	

40

Q.1 (16 points): Find the derivative, $\frac{dy}{dx}$. **(Simplify as much as possible)**

(i) $y = 2^{\sqrt{3+\cos(x)}}$

(ii) $y = e^{x^2}\sin(2x)$

(iii) $y = \cot^3\left(\cos\left(\frac{x}{3}\right)\right)$

(iv) $y = \log(x) + \log_x(10) + \log_x(x) + \log(10)$

Q.2 (6 points): Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Show that f is differentiable at $x = 0$.

Q.3 (4 points): Let $f(x)=\cos(2x-3)$. Find $f^{(2013)}(x)$, the 2013^{th} derivative of f.

Q.4 (4 points: If $g(x)=\left(x^2+2x+3\right)f(x)$, $f(0)=5$, and $\lim_{x\to 0}\frac{f(x)-5}{x}=4$.
Find $g'(0)$

Q.5 (5 points): Find the points on the curve: $y=x^3-11x+5$ at which the tangent line to the curve has the equation: $y+11=x$.

Q.6 (5 points): Let $f(x)=x^3-6x^2+ax+b$

If Rolle's Theorem holds with $c=2+\frac{1}{\sqrt{3}}$ on $[1,3]$, find a and b.

Q.7 (5 points): Let $f(x)=\begin{cases}\dfrac{1-\cos(x)}{x^3-x^2} & \text{if } x\neq 0\\ \text{k} & \text{if } x=0\end{cases}$. Find k so that f is continuous at $x=0$

Q.8 (5 points): Let $f(x)=\dfrac{x-1}{x+1}$

(a) Find the vertical and horizontal asymptotes of f, if any.

(b) Find $(f\circ f)(x)$ and its domain.

Q.9 (5 points): Find the point(s) $P(x,y)$ on the curve $y=x^2$ that is(are) closest to the point $(0,1)$.

Q.10 (16 points): Evaluate the following limits:

(i) $\lim_{x\to 4} \dfrac{x^3 - 7x^2 + 12x}{4 - x}$

(ii) $\lim_{x\to 0} \dfrac{\tan(7x) + 2x}{3x}$

(iii) $\lim_{x \to -\infty} \left(\sqrt{2x^2 - x} + 3x \right)$

(iv) $\lim_{x\to 3} \dfrac{-2}{(x-3)^3}$

Q.11 (8 points): Let $f(x) = xe^{x-x^2}$.

(a) Find the critical number(s) for f .

(b) Find the intervals on which f is increasing and/or decreasing.

(c) Find the local maximum and/or local minimum, if any.

Q.12 (8 points): (i) Show that if $y = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$, where $a > 0$, then $y' = \sqrt{a^2 - x^2}$

(ii) Show that if $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $y' = \frac{2}{1+x^2}$

Q.13 (10 points): Let $f(x)=\frac{x^2}{x^2+1}$.

(a) Find the intervals on which f is concave up or concave down.

(b) Find the inflection point(s) of f , if any.

(c) Sketch the graph of f showing all significant features.

Q.14 (3 points): Let $f(x)=e^{\frac{x}{2}}g(e^{-x})$ be a positive function. Given that g is differentiable and concave up, Show that f is concave up.