Prince Sultan University

Department of Mathematical Sciences

MATH 223 – Second Examination

21 May 2008

Time allowed: 100 minutes — Dr. Bahaa Eldin Abdalla

Maximum points: 40 points

1. Let $\mathbf{u} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - \mathbf{j} - 3\mathbf{k}$, and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. Verify that $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.
2. Find an equation of the plane passing through the point $(-2,3,4)$ and perpendicular to the line passing through the points $(4,-2,5)$ and $(0,2,4)$.
3. Show that the linear operator $T : R^2 \to R^2$ defined by $T(x_1,x_2) = (x_1 + 2x_2, -x_1 + x_2)$ is one-to-one and find T^{-1}.

4. Show that the transformation T defined by $T(x_1,x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear.

5. Show that the set V of all ordered triples of real numbers (x, y, z) with the operations $\oplus$ and $\odot$ defined by
$$(x,y,z) \oplus (x',y',z') = (x + x', y + y', z + z')$$
$$c \odot (x,y,z) = (cx, y, z)$$
is not a vector space.

6. (a) Show that R^2 is not a subspace of R^3.
   (b) Show that the set V of all polynomials of degree exactly = 2 is not a subspace of P_2.

7. In P_2 let $\mathbf{v}_1 = 2t^2 + t + 2$, $\mathbf{v}_2 = t^2 - 2t$, $\mathbf{v}_3 = 5t^2 - 5t + 2$, $\mathbf{v}_4 = -t^2 - 3t - 2$. Determine if the vector $\mathbf{u} = t^2 + t + 2$ belongs to span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.
8. Determine whether $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ are linearly independent vectors in M_{22}.
9. Determine bases for the subspace of R^3 that consistes of all vectors of the form (x, y, z) where $y = x + z$.
10. Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$. Show that $\langle \mathbf{u}, \mathbf{v} \rangle = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$ does not define an inner product on R^3.
