Prince Sultan University
Orientation Mathematics Program
Math223

Major 11

Fall Semester 091
Sunday, Dec. 13, 2009

Dr. Muhammad Islam Mustafa

Student Name: Time allowed: 60 minutes

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Q.1: *Write True(T) or False(F) for each of the following statements.* (5 pts)

1) The angle between the vectors $\mathbf{u}=(1,2)$ and $\mathbf{v}=(1,-1)$ is an acute angle. _____

2) The distance between the point (10, 0) and the line $3x+4y=0$ is 6. _____

3) The vectors $\mathbf{u}=(1,-1,3)$, $\mathbf{v}=(2,1,0)$, and $\mathbf{w}=(4,2,0)$ lie in the same plane. _____

4) The line $x=1-t, y=2t, z=2t$ and the plane $2x-4y+5z=0$ are perpendicular. _____

5) If $\|\mathbf{u}+\mathbf{v}\|=4$ and $\|\mathbf{u}-\mathbf{v}\|=2$, then $\mathbf{u}\cdot\mathbf{v}=3$. _____

6) If $\mathbf{u},\mathbf{v}\in R^n$ and A is a symmetric $n\times n$ matrix, then $A\mathbf{u}\cdot\mathbf{v}=\mathbf{u}\cdot A\mathbf{v}$. _____

7) The transformation $T(x_1,x_2)=(x_1+x_2, x_1-x_2, 1)$ is linear. _____

8) The standard matrix of the composition of a reflection about the y-axis followed by an orthogonal projection on the x-axis is $\begin{bmatrix}-1 & 0\\ 0 & 0\end{bmatrix}$. _____

9) If T is a linear transformation and $T(\mathbf{u})=(1,2), T(\mathbf{v})=(-1,1)$, then $T(2\mathbf{u}-\mathbf{v})=(3,3)$. _____

10) If $\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ are the standard basis of R^3 and T is a linear operator on R^3 such that $T(\mathbf{e}_1)=\begin{bmatrix}5\\1\\-1\end{bmatrix}, T(\mathbf{e}_2)=\begin{bmatrix}0\\0\\2\end{bmatrix}, T(\mathbf{e}_3)=\begin{bmatrix}0\\0\\3\end{bmatrix}$, then T is onto. _____

Q.2: Given that $\mathbf{u}=5\mathbf{i}-12\mathbf{j}$, $\mathbf{v}=\vec{PQ}$ where P is (1,0), $\|\mathbf{v}\|=26$ and $\mathbf{v}$ is in the opposite direction of $\mathbf{u}$. Find the terminal point Q of the vector $\mathbf{v}$. (3 pts)

Q.3: Prove that: If the vector **u** is orthogonal to the vector **v**, then $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$. (2 pts)

Q.4: Let $\mathbf{u} = (1,-2,3)$ and $\mathbf{v} = (1,0,1)$. Find (6 pts)

(a) The vector component of **u** along **v**.

(b) A vector that is perpendicular to the plane determined by **u** and **v**.

(c) An equation of the plane that contains the line $x = 2+t, y = 1-2t, z = 3t$ and is perpendicular to the plane $x + z = 5$

Q.5: Consider the linear operator $T: R^3 \to R^3$ defined by $\begin{cases} w_1 = x_1 + x_2 + x_3 \\ w_2 = 4x_1 + 5x_2 \\ w_3 = x_2 - 3x_3 \end{cases}$. (4 pts)

(a) Find the standard matrix of *T*.

(b) Show that *T* is one-to-one, and then find the formula of T^{-1}.