PRINCE SULTAN UNIVERSITY

MATH 223

MAJOR EXAM 3 13 JANUARY 2010

Start: 4:00 p.m.
End: 5:30 p.m.

Name: ____

I.D. ____________________________

MATH 223 / Major Exam 3 (Ch. 5, 6, 7) **Dr. Muhammad Islam Mustafa**

Student Name: Time allowed: 90 minutes

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Q.1: *Write True(T) or False(F) for each of the following statements.* (5 pts)

1) The set R^2, with the standard addition operation and the following scalar multiplication $k(x,y) = (kx, y)$, is a vector space. ____________

2) The set $W = \{p \in P_2 \mid p(1) = 0\}$ is a subspace of P_2 and $\dim(W) = 2$. ____________

3) The set $S = \{(1,0,2), (0,1,1), (-1,1,-1), (1,1,3)\}$ is a spanning set for R^3. ____________

4) If $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 2 & 0 & -2 \end{bmatrix}$, then $nullity(A) = 2$. ____________

5) If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors in an inner product space V and $\langle \mathbf{u}, \mathbf{v} \rangle = 0$, then $\mathbf{u}$ and $\mathbf{v}$ are linearly independent. ____________

6) Consider the vector space $C[-1,1]$, with the inner product $\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx.$

If $f(x) = x$ and $g(x) = x^2$, then f is orthogonal to g. ____________

7) Consider the inner product on R^2 generated by the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

If $\mathbf{u} = (1,0)$ and $\mathbf{v} = (0,1)$, then $d(\mathbf{u}, \mathbf{v}) = \sqrt{2}$. ____________

8) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix}$, then $\lambda = \frac{1}{8}$ is an eigenvalue of A^{-3}. ____________

9) If 2 is an eigenvalue of a matrix A, then 5 is an eigenvalue of the matrix $(A + 3I)$. ____________

10) The matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 3 & 4 \\ 2 & 4 & 7 \end{bmatrix}$ is diagonalizable. ____________

Q.2: Consider the set $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = (1,2,0), \mathbf{u}_2 = (0,3,2), \mathbf{u}_3 = (1,-1,-1)$. (5 pts)

(a) Show that *S* is a basis for R^3.

(b) Find the coordinate vector of $\mathbf{v} = (-1,4,7)$ relative to the basis *S*.

Q.3: Consider the matrix $A = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix}$. (6 pts)

(a) Find the eigenvalues and bases for the corresponding eigenspaces of the matrix *A*.

(b) Find the matrix *P* that diagonalize *A* and find P^{-1}.

(c) Find A^{10}.

Q.4: Each of the following is not an inner product on R^2. List the axioms that don't hold for each part. (4 pts)

(a) $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + u_1 v_2 + u_2 v_1 + u_2 v_2$

(b) $\langle \mathbf{u}, \mathbf{v} \rangle = u_1^{\,2} v_1^{\,2} + u_2^{\,2} v_2^{\,2}$