Prince Sultan University

Math 113
Major Exam 1
Second Semester, Term 122
Saturday, March 16, 2013

Time Allowed: 90 minutes

Student Name:_____ _____ _____

Student ID #: _____

Serial Class #:_____ Section #: 227 or 228

Instructor's Name: Dr. Aiman Mukheimer

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. Talking during the examination is NOT allowed.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.
7. This examination has 11 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3	23	
4,5	16	
6	20	
7,8	18	
9,10,11	23	
Total	100	

1. (6 points) Use Definition of the area to find an expression for the area under the graph of $f(x) = x^2 + \sqrt{1+2x}$ between $x = 4$ and $x = 7$ as a limit. **Do not evaluate the limit.**

2. (8 points) Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on [1, 8]. $\lim_{n\to\infty}[\frac{1}{\sqrt[3]{n^2}}(\frac{7}{\sqrt[3]{n+7}} + \frac{7}{\sqrt[3]{n+14}} + \frac{7}{\sqrt[3]{n+21}} + \frac{7}{\sqrt[3]{n+28}} + \cdots + \frac{7}{\sqrt[3]{n+7n}})]$

3. (9 points) Use a Riemann sum and a limit to compute the exact area under the curve of $y = x^3 + x$ on $[2,\ 4]$.

4. (8 points) Find a function f and a number m such that: $8 + \int_m^x \frac{f(t)}{t^2} dt = 2\sqrt{x}$ for all $x > 0$

5. (8 points) Verify that $f(x) = \sin(\sqrt[3]{x})$ is an odd function and use that fact to show that

$$0 \leq \int_{-2}^{3} \sin(\sqrt[3]{x})\, dx \leq 1$$

6. (20 points) Evaluate the following integrals: **(Show your work in details)**

i. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

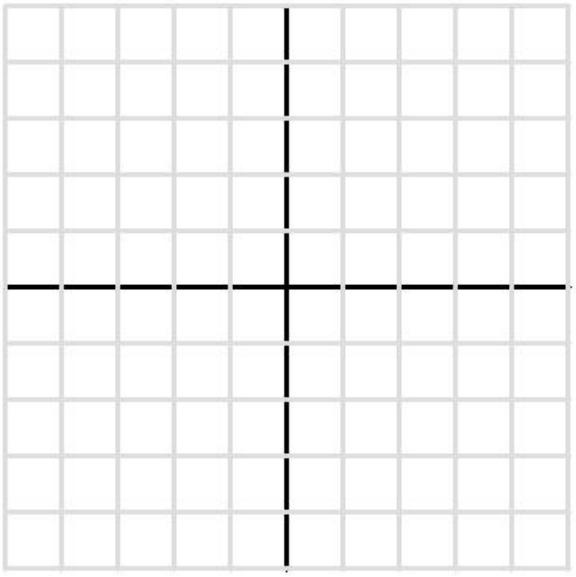
ii. $\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

iii. $\int \frac{x}{\sqrt{1-x^4}}\,dx$

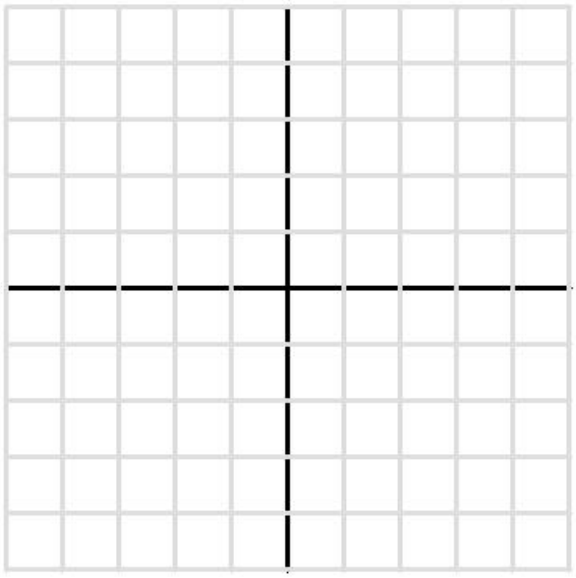
iv. $\int_0^{\frac{3\pi}{2}} \left|\sin x\right|\,dx$

7. (9 points) **Sketch** the region bounded by the curves below <u>and</u> **find** its area:

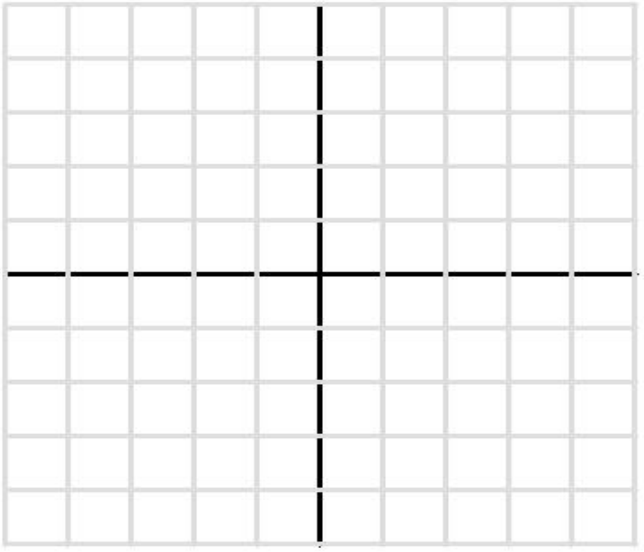
$y = x^2$, $y = -x^2$, $y = \dfrac{1}{x}$, $y = -1$ and $x = e$.



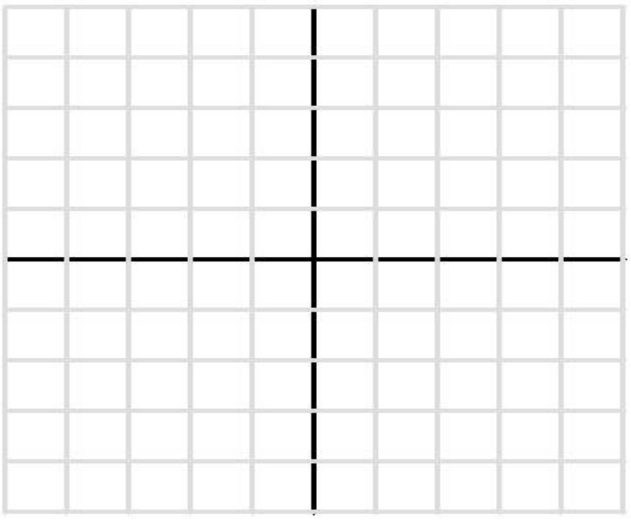
8. (9 point) **Sketch** the region bounded by the curves below <u>and</u> **compute** the volume of the solid formed by revolving the region about $y = 5$: $x + y = 3$, $y^2 + x = 3 + 2y$.



9. (10 point) Consider the volume resulting from revolving about the *y*-axis the region bounded by $y = \ln x$, $y = 1, y = 2,$ and $x = 1$. **Sketch** the region <u>and</u> **find** the volume.



10. (6 point) **Sketch** the region <u>and</u> **setup only** an integral to find the volume of the solid that results when the region enclosed by $y = \tan x, y = 0, x = \frac{\pi}{4}$, is revolved about $y = \frac{\pi}{2}$.



11. (7 points) Find the average value of f on [0,8].