Prince Sultan University

Orientation Mathematics Program

Math223

Major I

Fall Semester 091
Wednesday, November 04, 2009

Time Allowed: 50 minutes

Student Name: ________________________________

Student ID #: ________________________ Section #: ____________

Teacher's Name: ____________________________

<u>Important Instructions:</u>

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may **NOT borrow** a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be **NO talking** during the examination.
5. Your exam will be taken **immediately** if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. Provide an organized complete solution for each Question.

This examination has 12 problems. Make sure your paper has all these problems

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Q.1 *Write True(T) or False(F) for each of the following statements.* (4 pts)

1) If the augmented matrix of a linear system was reduced to $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, ____________

then the solution set for this system is $\{(2-z,1,z) \mid z \in R\}$.

2) $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 9 & 1 \end{bmatrix}$ is an invertible triangular matrix. ____________

3) Let A and B be two square matrices of the same size. The product AB is invertible if and only if both A and B are invertible. ____________

4) If a matrix A is symmetric, then the matrix A^2 is symmetric. ____________

5) $\begin{vmatrix} a & b & c \\ d & e & f \\ 3a & 3b & 3c \end{vmatrix} = 0$. ____________

6) If A is a 3×3 matrix and $\det(A) = 4$, then $\det(2A^{-1}) = \frac{1}{2}$. ____________

7) If A is a 3×3 triangular matrix, all its entries are nonnegative integers, and $\det(A) = 3$, then $tr(A) = 3$. ____________

8) $\lambda = 3$ is an eigenvalue of the system: $\begin{cases} x_1 + 2x_2 = \lambda x_1 \\ 2x_1 + x_2 = \lambda x_2 \end{cases}$. ____________

Q.2 Given that $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 7 & 3 \\ 1 & 0 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ -2 & 3 \\ -1 & 4 \end{bmatrix}$. (9 pts)

(a) Find $A^T B$

(b) Find A^{-1}

(c) Use A^{-1}, obtained in (b), to solve the system $\begin{cases} x + 3y + 2z = 1 \\ 2x + 7y + 3z = 2 \\ x + 6z = 3 \end{cases}$

(d) Use Cramer's rule to solve the same system in (c) for z only.

<u>Q3.</u> Use Gaussian elimination to find conditions that the b's must satisfy for the system to be consistent:

$$\begin{cases} x+y+2z=b_1 \\ x+z=b_2 \\ 2x+y+3z=b_3 \end{cases}$$

(4 pts)

<u>Q.4</u> Use row or column operations to prove that (3 pts)

$$\begin{vmatrix} b_1 & 2a_1+b_1 & c_1 \\ b_2 & 2a_2+b_2 & c_2 \\ b_3 & 2a_3+b_3 & c_3 \end{vmatrix} = -2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$