

Prince Sultan University
Department of Mathematics and Physical Sciences
Math215
Second Midterm Examination
Fall Semester 2012–2013
Exam duration: 90 minutes

Name:

Student ID:

Q.1 Let $\vec{u} = (2, 6, -7)$, $\vec{v} = (-1, -1, 8)$ and $k = 3$. If $(2, 14, 11) = k\vec{u} + l\vec{v}$, what is the value of l ?

Q.2 Determine whether $\vec{u} = (5, -2, 1)$, $\vec{v} = (4, -1, 1)$, $\vec{w} = (5, -4, 0)$ lie in the same plane positioned so their initial points coincide?

Q.3 Find an equation for the plane that contains the line $x = 2 + t$, $y = 1 - 2t$, $z = 3t$ and is perpendicular to the plane $x + z = 5$.

Q.4 Find $\vec{u} \cdot \vec{v}$ given that $\|u + v\| = 1$ and $\|u - v\| = 5$.

Q.5 Consider the linear operator $T : R^3 \rightarrow R^3$ defined by the equations

$$\begin{aligned}w_1 &= x_1 + 4x_2 - x_3 \\w_2 &= 2x_1 + 7x_2 + x_3 \\w_3 &= x_1 + 3x_2\end{aligned}$$

- a. Determine whether the linear operator T is one-to-one.
- b. If so, find the standard matrix for the inverse operator.
- c. Find a formula for $T^{-1}(w_1, w_2, w_3)$.

Q.6 Determine whether the vectors $\vec{u} = (2, -1, 3)$, $\vec{v} = (4, 1, 2)$, $\vec{w} = (8, -1, 8)$ span R^3 .

Q.7 Determine the dimension of and a basis for the solution space of the system

$$\begin{aligned}3x + y + z + w &= 0 \\5x - y + z - w &= 0\end{aligned}$$