Prince Sultan University

Department of Mathematical Sciences

MATH 223 – First Examination

31 March 2008

Time allowed: 75 minutes — Dr. Bahaa Eldin Abdalla

Maximum points: 40 points

1. (5 points) Solve the following system by Gaussian elimination.

$$-2y+3z=1$$
$$3x+6y-3z=-2$$
$$6x+6y+3z=5$$

2. (5 points) Let $A=\begin{bmatrix}3 & 0\\ -1 & 2\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{bmatrix}$. Compute $tr\left(B^T A^T\right)$.

3. (5 points) Let $A=\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$ and $p(x)=2x^2-x+1$. Find $p(A)$.

4. (3 points) Let A and B be two skew-symmetric matrices. Prove that $k(A+B)$ for any scalar *k* is also skew-symmetric.

5. (6 points) Let $A=\begin{bmatrix}3 & 3 & 1\\ 3 & 0 & -4\\ 1 & -3 & 5\end{bmatrix}$. Find each of the following.

   (a) $adj(A)$. (b) $\det(A)$. (c) A^{-1} using your results in a and b.

6. (6 points) Find the eigenvalues for the following system.

$$-2x+3y=\lambda x$$
$$x+4y=\lambda y$$

7. (4 points) Find the initial point of the unit vector **u** with terminal point $Q(3,0,-5)$ such that **u** is oppositely directed to **v** = $(3,4,0)$.

8. (6 points) Let **u** = $(2,-1,3)$, **v** = $(1,-3,4)$, and **w** = $(3,5,-4)$. Evaluate each of the following.

   (a) $\|3\mathbf{u}-5\mathbf{v}+\mathbf{w}\|$. (b) $\mathbf{u}\cdot(7\mathbf{v}-\mathbf{w})$
