

Chapter 19: Electric Charges, Forces, and Fields

Answers Conceptual Questions

4. Initially, the bits of paper are uncharged and are attracted to the comb by polarization effects. (See Figure 19-5 and the accompanying discussion.) When one of the bits of paper comes into contact with the comb, it acquires charge from the comb. Now the piece of paper and the comb have charge of the same sign, and hence there is a repulsive force between them.
8. No. If the ball is displaced slightly upward from the equilibrium position, the attractive electrostatic force will be larger than the gravitational force, which will displace the ball farther upward. Similarly, if the ball is displaced slightly downward, the gravitational force is now stronger than the electrostatic force, and the ball will move farther downward. Therefore, the equilibrium is unstable.
10. The proton can be moving in any direction at all relative to the direction of the electric field. On the other hand, the direction of the proton's acceleration *must* be in the same direction as the electric field.
18. No. The electric flux through a surface depends on the total charge enclosed by the surface, but is completely independent of the location of the enclosed charges.

Solutions to Problems and Conceptual Exercises

2. An electrically neutral object is given a negative charge.

Each electron carries both charge and mass, so adding or subtracting electrons will change the object's mass.

Solution: 1. (a) Giving the object a negative charge requires the addition of electrons, which have mass. We conclude that the object's mass will increase as a result of being charged.

2. (b) The best explanation is I. To give the object a negative charge we must give it more electrons, and this will increase its mass. Statement II is false, and statement III fails to distinguish between charge and mass conservation.

Likewise, giving an object a positive charge requires the removal of electrons, decreasing the object's mass.

7. A large number of carbon atoms are collected together in a system.

Each carbon atom has 6 electrons, and each electron carries a charge equal to $-e = -1.60 \times 10^{-19} \text{ C}$. We need only multiply the number of electrons by the amount of charge carried by each.

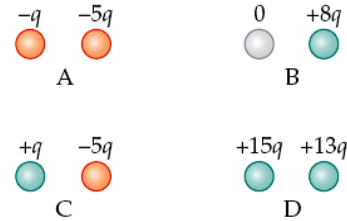
Solution: Multiply the number of electrons by $-e$:

$$Q = N(-e) = (6nN_A)(-e)$$

$$= 6(2 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1})(-1.60 \times 10^{-19} \text{ C}) = \boxed{-1 \times 10^6 \text{ C}}$$

This is a huge amount of charge, but you must keep in mind that carbon atoms are electrically neutral, so that there is an equal amount of positive charge in the nuclei and the net charge of the carbon atoms is zero.

11. Four pairs of conducting spheres, all with the same radius, are shown in the figure at right, along with their initial charges. The spheres in each pair are now brought into contact, allowing charge to transfer between them.



Charge will be transferred until the two spheres in each pair have the same charge. Sum the charge on each pair and divide by two to determine the net charge remaining on each sphere after the transfer. Compare that value to the initial charge to determine the amount of charge transferred.

Solution: For pair A we note that the total charge is $-6q$ so that $-3q$ will remain on each sphere after they are brought into contact. That means that $2q$ of charge is transferred. By a similar reasoning we find that $4q$ is transferred for pair B, $3q$ is transferred for pair C, and charge q is transferred for pair D. The ranking is $\boxed{D < A < C < B}$.

If a pair had charges $-5q$ and $+5q$, the net charge is zero, and charge $5q$ would be transferred if the two spheres were brought into contact.

16. An insulating sphere has a uniform surface charge density, and exerts a force on a point charge that is placed nearby but outside the sphere. The insulating sphere is then replaced by an otherwise identical conducting sphere.

Consider the behavior of insulating and conducting materials when answering the conceptual question.

Solution: 1. (a) The magnitude of the force between the point charge and the conducting sphere will be less than the force between the point charge and an insulating sphere. To see this, first note that both the sphere and the point charge are positive. Therefore, the positive charge on the sphere will move as far away from the point charge as possible (as opposed to the uniform charge on an insulator, which cannot move). The larger distance between the charges will reduce the repulsive force between the conducting sphere and the insulating sphere according to Coulomb's law.

2. (b) The best explanation is III. The charge on a conducting sphere will move as far away as possible from the point charge. This results in a reduced force. Statement I is false, and statement II ignores the effect of the redistribution of charge on the conducting sphere due to the nearby point charge.

If the sphere were charged positively and the point charge were negative, then positive charges would collect on the near side of the conducting sphere and the force would be greater than it would be for an insulating sphere.

20. Two charges of unequal magnitude exert an electrostatic force on each other.

Use Coulomb's law (equation 19-5) to find the magnitude of the force between the two charges.

Solution: 1. (a) Apply equation 19-5 directly:

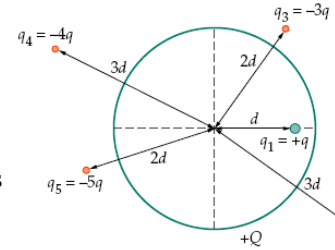
$$F = k \frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(3.13 \times 10^{-6} \text{ C})(4.47 \times 10^{-6} \text{ C})}{(0.255 \text{ m})^2} =$$

2. (b) The magnitude of the electrostatic force depends upon the product of the charges of both particles, so the negative charge experiences a force magnitude that is the same as that experienced by the positive charge.

The forces experienced by the two charges must also be the same magnitude and opposite in direction in order to be consistent with Newton's Third Law.

24. Five point charges, $q_1 = +q$, $q_2 = +2q$, $q_3 = -3q$, $q_4 = -4q$, and $q_5 = -5q$, are placed in the vicinity of an insulating spherical shell with a charge $+Q$ distributed uniformly over its surface.

Use Coulomb's law (equation 19-5) to determine the magnitudes of the forces exerted on each of the charges. Treat the charge on the sphere as if it were a point charge at the center of the sphere. A charge inside of the shell would experience no net force.



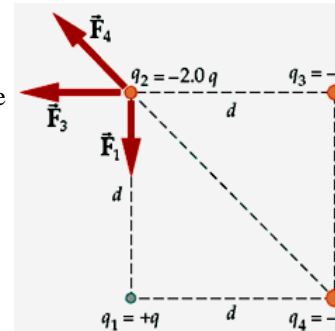
Solution: The magnitude of the force exerted on each of the charges is proportional to the magnitude of its charge and inversely proportional to the square of the distance from the charge to the center of the sphere. If $F_0 = kQq/d^2$, we can find that

$F_1 = 0$, $F_2 = \frac{2}{9}F_0$, $F_3 = -\frac{3}{4}F_0$, $F_4 = -\frac{4}{9}F_0$, and $F_5 = -\frac{5}{4}F_0$. The ranking of the magnitudes (ignoring the signs) is thus $F_1 < F_2 < F_4 < F_3 < F_5$.

Calculus can be used to prove $F_1 = 0$, but it is very difficult. An easier approach is to use Gauss' law (section 19-7), which indicates that the electric field (and thus the force on q_1) is zero everywhere inside the shell.

31. Four charges are situated at the corners of a square as shown in the diagram at right.

The force on charge q_2 is a vector sum of the forces from the other three charges. Let q_2 be at the origin and q_3 be on the positive x -axis. Use Coulomb's law (equation 19-5) to find the vector sum of the three forces, from which we can find the magnitude and direction of the net electrostatic force on q_2 .



Solution: 1. Find \vec{F}_1 : $\vec{F}_1 = -\frac{k|q_1||q_2|}{d^2}\hat{y} = -\frac{kq(2.0q)}{d^2}\hat{y} = -\frac{2.0kq^2}{d^2}\hat{y}$

2. Find \vec{F}_3 : $\vec{F}_3 = -\frac{k|q_2||q_3|}{d^2}\hat{x} = -\frac{k(2.0q)(3.0q)}{d^2}\hat{x} = -\frac{6.0kq^2}{d^2}\hat{x}$

3. Find \vec{F}_4 : $\vec{F}_4 = \frac{k|q_2||q_4|}{(\sqrt{2}d)^2}\left(-\frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}}\right) = \frac{k(2.0q)(4.0q)}{(\sqrt{2}d)^2}\left(-\frac{\hat{x}}{\sqrt{2}} + \frac{\hat{y}}{\sqrt{2}}\right) = \frac{2.0\sqrt{2}kq^2}{d^2}(-\hat{x} + \hat{y})$

4. Find the vector sum of the three forces:

$$\begin{aligned}\vec{\mathbf{F}}_{\text{net}} &= \left(-\frac{2.0\sqrt{2}kq^2}{d^2} - \frac{6.0kq^2}{d^2} \right) \hat{\mathbf{x}} + \left(\frac{2.0\sqrt{2}kq^2}{d^2} - \frac{2.0kq^2}{d^2} \right) \hat{\mathbf{y}} \\ &= \frac{2.0kq^2}{d^2} \left[-(\sqrt{2} + 3.0) \hat{\mathbf{x}} + (\sqrt{2} - 1) \hat{\mathbf{y}} \right] \\ &= \frac{2.0(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(2.4 \times 10^{-6} \text{ C})^2}{(0.33 \text{ m})^2} \left[-(\sqrt{2} + 3.0) \hat{\mathbf{x}} + (\sqrt{2} - 1) \hat{\mathbf{y}} \right] \\ \vec{\mathbf{F}}_{\text{net}} &= (-4.2 \text{ N}) \hat{\mathbf{x}} + (0.39 \text{ N}) \hat{\mathbf{y}}\end{aligned}$$

5. Find the direction of $\vec{\mathbf{F}}_{\text{net}}$ from the $+x$ axis:

$$\theta = \tan^{-1} \frac{F_{\text{net},y}}{F_{\text{net},x}} = \tan^{-1} \frac{0.39 \text{ N}}{-4.1 \text{ N}} = -5.4^\circ + 180^\circ = \boxed{174.6^\circ}$$

6. Find the magnitude of $\vec{\mathbf{F}}_{\text{net}}$:

$$F_{\text{net}} = \sqrt{(-4.2 \text{ N})^2 + (0.39 \text{ N})^2} = \boxed{4.2 \text{ N}}$$

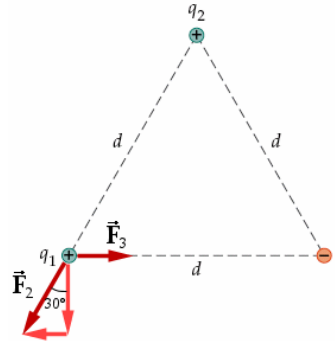
$\vec{\mathbf{F}}_3$ is the largest of the three force, three times bigger than $\vec{\mathbf{F}}_1$ and over twice as big as $\vec{\mathbf{F}}_4$. That is why $\vec{\mathbf{F}}_{\text{net}}$ points mainly in the same direction as $\vec{\mathbf{F}}_3$.

35. Three charges are arranged as indicated in the figure and exert electrostatic forces on each other.

Use Coulomb's law (equation 19-5) to find the electric forces on q_1 due to the other two charges, then find the vector sum of those two forces using the component method of vector addition. Let q_1 be at the origin and q_3 be on the positive x -axis.

Solution: 1. (a) Use Coulomb's law to find the components of $\vec{\mathbf{F}}_2$:

$$\vec{\mathbf{F}}_2 = \frac{k|q_1||q_2|}{d^2} \left[(-\sin 30^\circ) \hat{\mathbf{x}} + (-\cos 30^\circ) \hat{\mathbf{y}} \right]$$



2. Find the magnitudes of the components:

$$\begin{aligned}\vec{\mathbf{F}}_2 &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(2.1 \times 10^{-6} \text{ C})(6.3 \times 10^{-6} \text{ C})}{(0.0435 \text{ m})^2} \left[-\frac{1}{2} \hat{\mathbf{x}} - \frac{\sqrt{3}}{2} \hat{\mathbf{y}} \right] \\ &= (-31 \text{ N}) \hat{\mathbf{x}} + (-54 \text{ N}) \hat{\mathbf{y}}\end{aligned}$$

3. Use Coulomb's law to find the components of $\vec{\mathbf{F}}_3$:

$$\begin{aligned}\vec{\mathbf{F}}_3 &= \frac{k|q_1||q_3|}{d^2} \hat{\mathbf{x}} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(2.1 \times 10^{-6} \text{ C})(0.89 \times 10^{-6} \text{ C})}{(0.0435 \text{ m})^2} \hat{\mathbf{x}} \\ &= (8.9 \text{ N}) \hat{\mathbf{x}}\end{aligned}$$

4. Add the components to find $\vec{\mathbf{F}}_{\text{net}}$:

$$\begin{aligned}\vec{\mathbf{F}}_{\text{net}} &= \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = (-31 + 8.9 \text{ N}) \hat{\mathbf{x}} + (-54 \text{ N}) \hat{\mathbf{y}} \\ &= (-22 \text{ N}) \hat{\mathbf{x}} + (-54 \text{ N}) \hat{\mathbf{y}}\end{aligned}$$

5. Find the direction of $\vec{\mathbf{F}}_{\text{net}}$, measured counterclockwise from the $+x$ -axis:

$$\theta = \tan^{-1} \left(\frac{F_{\text{net},y}}{F_{\text{net},x}} \right) = \tan^{-1} \left(\frac{-54 \text{ N}}{-22 \text{ N}} \right) = 68^\circ + 180^\circ = \boxed{248^\circ}$$

6. Find the magnitude of \vec{F}_{net} : $F_{\text{net}} = \sqrt{F_{\text{net},x}^2 + F_{\text{net},y}^2} = \sqrt{(-22 \text{ N})^2 + (-54 \text{ N})^2} = \boxed{58 \text{ N}}$

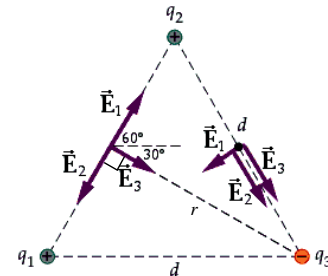
7. (b) If the distance d were doubled, the magnitude of the electrostatic force would be decreased by a factor of 4 and the direction would be unchanged.

If q_3 were instead a positive charge, \vec{F}_3 would point toward the left, the magnitude of the net force would be larger (67 N), and the angle θ would point closer to the $-\hat{x}$ direction (233°).

48. Three charges are positioned as shown at right.

Each of the three charges produces its own electric field that surrounds it. The total electric field at any point is the vector sum of the fields from each charge. Use equation 19-10 and the component method of vector addition to find the magnitude electric field at the points indicated in the problem statement.

Let q_1 be at the origin and q_3 be on the positive x -axis.



Solution: 1. (a) At a point halfway between

charges q_1 and q_2 the vectors \vec{E}_1

and \vec{E}_2

cancel one another. The remaining contri-

bution comes from q_3 . Find the distance r

from q_3 to the midpoint of the opposite side:

$$\begin{aligned} r^2 + (d/2)^2 &= d^2 \\ r &= \sqrt{3d^2/4} \\ &= \sqrt{3(0.0295 \text{ m})^2/4} \\ r &= \underline{\underline{0.02555 \text{ m}}} \end{aligned}$$

2. Apply equation 19-10 to find E_3 : $E_3 = \frac{kq_3}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.00 \times 10^{-6} \text{ C})}{(0.02555 \text{ m})^2} = \boxed{6.89 \times 10^7 \text{ N/C}}$

3. (b) At this location, the electric fields from q_2 and q_3 add, and the resulting field points toward q_3 . The field due to q_1 will have the same magnitude as found in part (a) and will be perpendicular to the combined fields of q_2 and q_3 . The vector sum of the electric fields from all three charges will have a magnitude greater than that found in part (a).

4. (c) Find the components of \vec{E}_1 : $\vec{E}_1 = \frac{k|q_1|}{3d^2/4} (\cos 30^\circ \hat{x} + \sin 30^\circ \hat{y}) = \frac{k|q_1|}{d^2} \left(\frac{2\sqrt{3}}{3} \hat{x} + \frac{2}{3} \hat{y} \right)$

5. Find the components of \vec{E}_2 : $\vec{E}_2 = \frac{k|q_2|}{(d/2)^2} (\cos 60^\circ \hat{x} - \sin 60^\circ \hat{y}) = \frac{k|q_2|}{d^2} (2\hat{x} - 2\sqrt{3}\hat{y})$

6. Find the components of \vec{E}_3 : $\vec{E}_3 = \frac{k|q_3|}{(d/2)^2} (\cos 60^\circ \hat{x} - \sin 60^\circ \hat{y}) = \frac{k|q_3|}{d^2} (2\hat{x} - 2\sqrt{3}\hat{y})$

7. Let $|q_1| = |q_2| = |q_3| = q$ and find the vector sum:

$$\begin{aligned} \vec{E}_{\text{net}} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\ \vec{E}_{\text{net}} &= \frac{kq}{d^2} \left[\left(4 + \frac{2\sqrt{3}}{3} \right) \hat{x} + \left(\frac{2}{3} - 4\sqrt{3} \right) \hat{y} \right] \end{aligned}$$

8. Determine the magnitude of E_{net} :

$$E_{\text{net}} = \frac{kq}{d^2} \sqrt{\left(4 + \frac{2\sqrt{3}}{3}\right)^2 + \left(\frac{2}{3} - 4\sqrt{3}\right)^2}$$

$$= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(5.00 \times 10^{-6} \text{ C})}{(0.0295 \text{ m})^2} (8.110)$$

$$E_{\text{net}} = \boxed{4.19 \times 10^8 \text{ N/C}} = 419 \text{ MN/C}$$

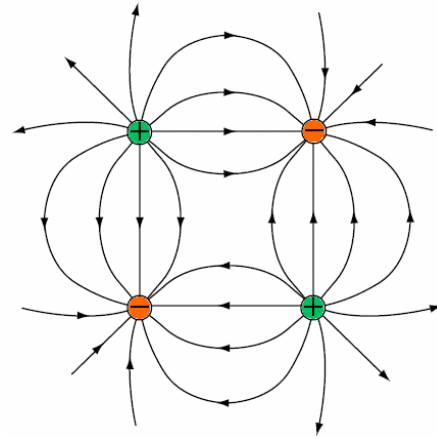
As expected, the field is larger at the point midway between q_2 and q_3 , about six times larger in magnitude than at the point midway between q_1 and q_2 .

57. Four charges of equal magnitude but alternating signs are arranged at the corners of a square as shown in the diagram at the right.

Electric field lines diverge away from positive charges and converge toward negative charges. The number of lines of force entering or leaving a charge is proportional to the magnitude of the charge.

Solution: The lines of force are sketched at right.

An electric field diagram can reveal a wealth of information in a concise fashion when it is drawn correctly.



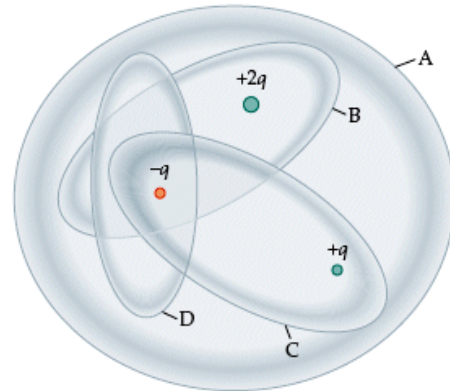
63. Four Gaussian surfaces are shown at right.

Apply Gauss's law to answer the conceptual questions.

Solution: The electric flux through each surface is proportional to the amount of charge enclosed by that surface. Surface A encloses charge $+2q$, surface B encloses charge $+q$, surface C encloses zero net charge, and surface D encloses charge $-q$. The ranking of these surfaces in order of increasing electric flux, starting with the most negative, is therefore:

$$\boxed{D < C < B < A}$$

Even though the electric flux through surface C is zero, it does not mean that $E = 0$ everywhere across surface C. In fact, there is a substantial field at every part of surface C, but there are just as many electric field lines entering the surface as there are leaving the surface.



80. A point charge situated at the origin produces an electric field that completely surrounds it.

Use the definition of the electric field (equation 19-10) to find the magnitude of the charge that creates the stipulated electric field. Because the electric field points toward the origin and therefore toward the charge, the sign of the charge must be negative.

Solution: Solve equation 19-10 for q , assuming the charge is negative:

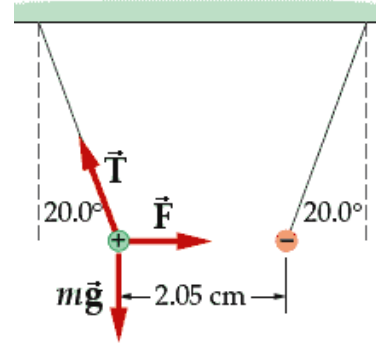
$$|q| = \frac{E r^2}{k} = \frac{(36,000 \text{ N/C})(0.75 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2} = 2.3 \times 10^{-6} \text{ C}$$

$$q = \boxed{-2.3 \mu\text{C}}$$

If the sign of the charge had been positive, the field would have pointed in the $-\hat{x}$ direction.

94. The configuration of the charged balls and the force vectors on the left ball are depicted in the diagram at right.

The magnitude of the electric force, the tension in each thread, and the magnitude of the charge on each ball are the same for both balls. So, we will solve for these quantities with respect to the ball on the left. Apply Newton's Second Law in the horizontal and vertical directions to the left ball and solve for the magnitude of the electric force F , the string tension T , and the charge q on each ball.



Solution: 1. (a) Write Newton's Second Law in each direction for the left ball:

$$\sum F_x = T \sin 20.0^\circ - F = 0$$

$$\sum F_y = T \cos 20.0^\circ - mg = 0$$

2. Solve the x equation for F :

$$F = T \sin 20.0^\circ = \left(\frac{mg}{\cos 20.0^\circ} \right) \sin 20.0^\circ = mg \tan 20.0^\circ$$

$$= (1.4 \times 10^{-4} \text{ kg})(9.81 \text{ m/s}^2) \tan 20.0^\circ = \boxed{5.0 \times 10^{-4} \text{ N}} = 0.50 \text{ mN}$$

3. (b) Solve the y equation for T :

$$T = \frac{mg}{\cos 20.0^\circ} = \frac{(1.4 \times 10^{-4} \text{ kg})(9.81 \text{ m/s}^2)}{\cos 20.0^\circ} = \boxed{1.5 \times 10^{-3} \text{ N}} = 1.5 \text{ mN}$$

4. (c) Use the result of part (a) to $F = \frac{kq^2}{d^2}$ find q :

$$q = d \sqrt{\frac{F}{k}} = (0.0205 \text{ m}) \sqrt{\frac{5.0 \times 10^{-4} \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} = \boxed{4.8 \times 10^{-9} \text{ C}} = 4.8 \text{ nC}$$

If the mass of each ball were to double, both F and T would double as well, but the charge on each would only need to increase by $\sqrt{2}$ in order for the thread angle to remain 20.0° with the vertical.