Prince Sultan University

Department of Mathematics and Physical Sciences

Math 113
First Midterm Exam
Semester II, Term 132
Thursday, March 13, 2014

Time Allowed: 80 minutes

Answer Key

Name:		
Student Number:		
Instructor's Name:	Nabil Mlaiki,	Jehad Alzabut
Section:	222 223	220 221

Statement of Ethics:

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

A deal: You have to

1. Show details of your all work,
2. Use the calculator upon proper calculations.

Total/60:

Total/20:

Problem 1. (5+7+3 points)

a) Evaluate the Riemann sum for $f(x) = x^2 + x - 2$, $0 \le x \le 2$, with four subintervals, taking the sample points to be left endpoints. (1)

0, 0.5, 1, 1.5, 2

$f(x) = x^2 + x - 2$

$n = 4, \quad \Delta x = 0.5$

$$L_4 = 0.5\left[f(0) + f(0.5) + f(1) + f(1.5)\right] \quad (2)$$

$$= 0.5\left[(-2) + \left(-\tfrac{5}{4}\right) + 0 + \tfrac{7}{4}\right] = \boxed{-\tfrac{3}{4}} \quad (2)$$

b) Use the limit definition of Riemann sum for the definite integral to evaluate $\int_0^2 (x^2 + x - 2)dx$. (1)

$\Delta x = \frac{2}{n}$

$x_i = \frac{2i}{n}$

$$\int_0^2 (x^2+x-2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\cdot\frac{2}{n} \quad (1)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^2 + \left(\frac{2i}{n}\right) - 2\right]\cdot\frac{2}{n} = \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} i - \sum_{i=1}^{n} 2\right] \quad (2)$$

$$= \lim_{n\to\infty}\left[\frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{4}{n^2}\,\frac{n(n+1)}{2} - \frac{2}{\cancel{n}}\cdot 2\cancel{n}\right] \quad (2)$$

$$= \frac{16}{6} + \frac{4}{2} - 4$$

$$= \frac{16}{6} + 2 - 4 = \frac{16}{6} - 2 = \frac{4}{6} = \boxed{\frac{2}{3}} \quad (1)$$

c) Use the *Fundamental Theorem of Calculus* to evaluate $\int_0^2 (x^2 + x - 2)dx$ and check your answer in part (b).

$$\int_0^2 (x^2+x-2)\,dx = \int_0^2 x^2\,dx + \int_0^2 x\,dx - 2\int_0^2 dx = \frac{x^3}{3}\Big|_0^2 + \frac{x^2}{2}\Big|_0^2 - 2x\Big|_0^2 \quad (2)$$

$$= \left(\frac{8}{3} - 0\right) + \left(\frac{4}{2} - 0\right) - (4 - 0) =$$

$$= \frac{8}{3} + 2 - 4 = \frac{8}{3} - 2 = \boxed{\frac{2}{3}} \quad (1)$$

Problem 2. (4 points) Verify the inequality $\int_0^1 e^x \cos x\,dx \le e-1$. 1

Start by writing the identity: $\cos x \le 1$. Then

1 $\Rightarrow e^x \cdot \cos x \le e^x \Rightarrow \int_0^1 e^x \cos x\,dx \le \int_0^1 e^x\,dx = e^1 - e^0 = \boxed{e-1}$ 2

$\therefore \int_0^1 e^x \cos x\,dx \le e-1.$ //

Problem 3. (4+6 points) Evaluate the integrals:

a) $\int (3\sin x + 4)^5 \cos x\,dx = \int u^5 \cdot \cancel{\cos x} \cdot \frac{du}{3\cancel{\cos x}} = \frac{1}{3}\int u^5\,du$ 1

Let $u = 3\sin x + 4$

$du = 3\cos x\,dx$ 1

$= \frac{1}{3}\,\frac{u^6}{6} + C$ 1

$= \frac{1}{18}(3\sin x + 4)^6 + C.$ 1 //

or

$u = \cos 2x$

$dv = e^{-x}dx$

$\downarrow$

$\vdots$

b) $\int \underbrace{e^{-x}}_{u} \underbrace{\cos(2x)dx}_{dv}$

Let $u = e^{-x} \rightarrow du = -e^{-x}dx$

$dv = \cos 2x \rightarrow v = \frac{1}{2}\sin 2x$ 1

$\int e^{-x}\cos(2x)dx = \frac{1}{2}e^{-x}\sin 2x + \frac{1}{2}\int e^{-x}\sin 2x\,dx$ 1

Let $u = e^{-x} \rightarrow du = -e^{-x}dx$

$dv = \sin 2x \rightarrow v = -\frac{1}{2}\cos 2x$ 1

$\int e^{-x}\cos(2x)dx = \frac{1}{2}e^{-x}\sin 2x + \frac{1}{2}\left[-\frac{1}{2}e^{-x}\cos 2x - \frac{1}{2}\int e^{-x}\cos 2x\,dx\right]$ 1

$\therefore \frac{5}{4}\int e^{-x}\cos 2x\,dx = \frac{1}{2}e^{-x}\sin 2x - \frac{1}{4}e^{-x}\cos 2x + C$ 2

$\int e^{-x}\cos 2x\,dx = \frac{4}{5}\left[\frac{1}{2}e^{-x}\sin 2x - \frac{1}{4}e^{-x}\cos 2x + C\right]$ //

Problem 4. (6 points) Find a value of c as guaranteed by *Mean Value Theorem* for $f(x) = \dfrac{1}{3-2x}$ on the interval $[-1,+1]$.

By MVT $\Rightarrow f(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$ 1

$$f(c) = \frac{1}{2}\int_{-1}^{1}\frac{1}{3-2x}\,dx, \quad \begin{vmatrix} u = 3-2x \\ du = -2dx \\ x: -1 \longrightarrow 1 \\ u: 5 \longrightarrow 1 \end{vmatrix} \Rightarrow \frac{1}{3-2c} = \frac{1}{2}\int_5^1 \frac{1}{u}\cdot\frac{du}{-2} = 2$$

1 1 1

$$= \frac{1}{4}\int_1^5 \frac{1}{u}\,du = \frac{1}{4}\ln u\Big|_1^5 = \frac{1}{4}\ln 5 \Rightarrow \frac{1}{3-2c} = \frac{\ln 5}{4} \Rightarrow$$ 2

$$3-2c = \frac{4}{\ln 5} \Rightarrow 3 - \frac{4}{\ln 5} = 2c \Rightarrow \boxed{c = 0.257} \in [-1,1]$$

Problem 5. (5+7 points) Let R be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Find

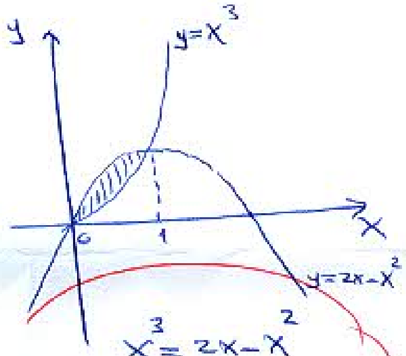
a) The area of R.

$$\text{Area} = \int_0^1 (2x - x^2 - x^3)\,dx = x^2\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 - \frac{x^4}{4}\Big|_0^1$$ 1

2

$$= (1-0) - \left(\frac{1}{3}-0\right) - \left(\frac{1}{4}-0\right)$$

1 $$= 1 - \frac{1}{3} - \frac{1}{4} = \boxed{\frac{5}{12}}$$



$x^3 = 2x - x^2$

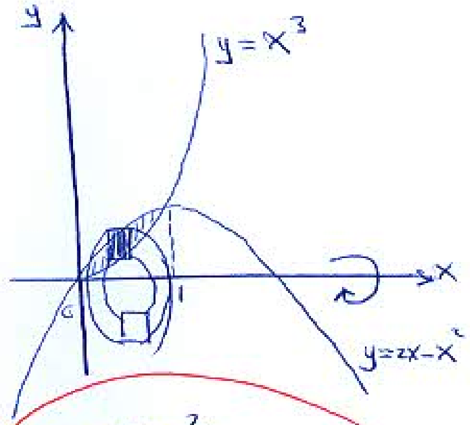
1 $x^3 + x^2 - 2x = 0$

$x = 0, x = 1, \cancel{x = -2}$

b) The volume obtained by rotating R about $x-$axis.

$$V = \int_a^b A(x)\,dx$$ 4

$$= \int_0^1 \pi\left((2x-x^2)^2 - (x^3)^2\right)dx$$



$A(x) = \pi r^2$

$= \pi r_o^2 - \pi r_i^2$

$= \pi(2x-x^2)^2 - \pi(x^3)^2$

$$= \int_0^1 \pi\left(4x^2 - 4x^3 + x^4 - x^6\right)dx$$

3 $$= \pi\left[\frac{4x^3}{3}\Big|_0^1 - \frac{4x^4}{4}\Big|_0^1 + \frac{x^5}{5}\Big|_0^1 - \frac{x^7}{7}\Big|_0^1\right]$$

$$= \boxed{\frac{41}{105}\pi}$$

Problem 6. (5 points) If f is a continuous function such that $\int_1^x f(t)dt = (x-1)e^{2x} + \int_1^x e^{-t} f(t)dt$ for all x, find an explicit formula for $f(x)$. Hint: derive then solve for $f(x)$.

Take the derivative: $f(x) = e^{2x} + 2(x-1)e^{2x} + e^{-x}f(x)$ 3

$\Rightarrow f(x) - e^{-x}f(x) = e^{2x} + 2xe^{2x} - 2e^{2x} = 2xe^{2x} - e^{2x}$ 2

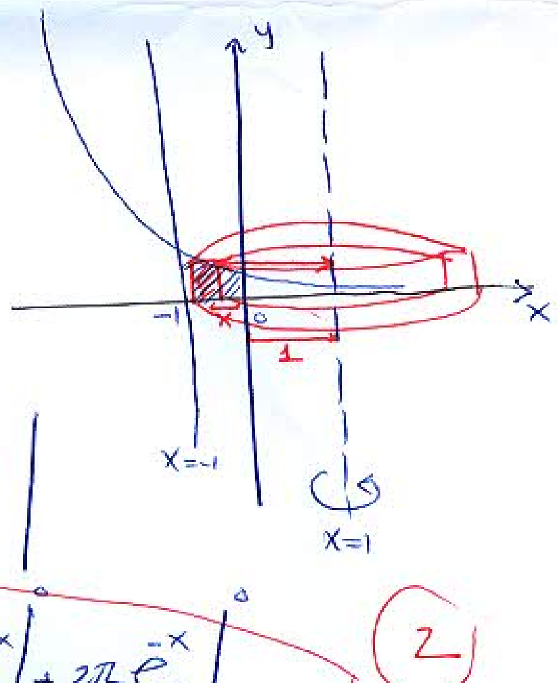
$$\therefore \boxed{f(x) = \frac{2xe^{2x} - e^{2x}}{1 - e^{-x}}}$$

Problem 7. (8 points) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = e^{-x}$, $y = 0$, $x = -1$, $x = 0$, about the line $x = 1$.

$$V = \int_a^b 2\pi\, r(x)\, f(x)\, dx$$

3

$$= \int_{-1}^{0} 2\pi \underbrace{(1-x)}_{u} \underbrace{e^{-x}dx}_{dv} = 2$$



1

$u = 1 - x \longrightarrow du = -dx$

$dv = e^{-x}dx \longrightarrow v = -e^{-x}$

2

$$= 2\pi\left(-(1-x)e^{-x}\Big|_{-1}^{0} - \int_{-1}^{0} e^{-x}dx\right) = -2\pi(1-x)e^{-x}\Big|_{-1}^{0} + 2\pi e^{-x}\Big|_{-1}^{0}$$

2

$$= -2\pi(1-2e) + 2\pi(1-e)$$

$$= -\cancel{2\pi} + \cancel{2\pi} + 4\pi e - 2\pi e$$

$$= 2\pi e$$