Department of Mathematical Sciences

Dr. Muhammad Islam Mustafa

MATH 221

Major Exam 1

Student Name: _____ Student ID #: _____

Section #: _____ Time allowed: 75 minutes

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Q1. Consider the function $f(x) = e^x + \sin x$.

a) Find the third Taylor polynomial $P_3(x)$ for the function $f(x)$ about $x_0 = 0$.

b) Use $P_3(0.5)$ to approximate $f(0.5)$. Find an upper bound for the absolute error $|f(0.5) - P_3(0.5)|$ using the error formula.

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Q2. Consider the equation $x^3 - x - 2 = 0$.

a) Use the Intermediate Value Theorem to show that this equation has a solution in the interval [1, 2].

b) Use the Bisection method on the interval [1, 2] to find p_3.

c) Find the number of iterations required in using the Bisection method to approximate the solution of this equation accurate to within 10^{-3}.

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Q3. a) Show that the fixed point of the function $g(x) = \sqrt[3]{x+2}$ is a solution for the equation $x^3 - x - 2 = 0$.

b) Use the Fixed Point Theorem to prove that $g(x)$ has a unique fixed point on [1, 2].

c) Begin with $p_0 = 1.5$ and find an approximation to this fixed point accurate to within 10^{-3}.

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Q4. Use Newton's method, with $p_0 = 1.5$, to approximate the solution of the equation $x^3 - x - 2 = 0$ accurate to within 10^{-3}.

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Q5. Find an approximation to $\sqrt{5}$ correct to within 10^{-2} using the method of false position.
(Hint: consider the function $f(x) = x^2 - 5$ on the interval [2, 3])