

1) Consider the integro-differential equation:

$$\phi'(t) + \phi(t) = \int_0^t \sin(t-s)\phi(s)ds, \quad \phi(0) = 1$$

- a) Solve the above equation by using Laplace transform.
- b) Transform the above integro-differential equation into an initial value problem by differentiating, then solve it and verify your solution in part (a).

2) Find the Laplace transform for the functions:

- i) $f(t) = e^{-2t} \sin t$.
- ii) $g(t) = u_2(t) \cos(2t - 4)$.

3) Find the Laplace inverse of the following function once by partial fractions **and** once by the convolution Theorem.:

$$F(s) = \frac{1}{(s^2 + 1)(s^2 + 4s + 5)}.$$

4) Solve the initial value problem by using Laplace transforms **then** verify by using the method of undetermined coefficients:

$$\frac{d^2 y}{dt^2} + y = e^{-2t} \sin t, \quad y(0) = 0, \quad y'(0) = 0$$