1)Consider the differential equation:

$$Ly(x) = (2x+1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0. \tag{1}$$

a) Verify that $y_1(x) = e^{2x}$ is a solution of (1) and then use the reduction of order to find a second linearly independent solution. Afterthen, find the Wronskian of the the two solutions.

b) Find the general solution of the nonhomogenous equation:

$$Ly(x) = e^{2x}(2x+1)^2$$

. (Hint: Use the method of variation of parameters to find the particular solution).

2)a) Find the general solution of:

$$\frac{d^2y}{dt^2} + y = 4sint. \tag{2}$$

b) Solve the initial value problem:

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 4sin(lnx),\ x > 0,\ ,\ y(1) = 0,\ y'(1) = 1. \tag{3}$$

3) a) Discuss the type of singularity at infinity for the Legendre differential equation:

$$(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0,\ \alpha \in \mathbb{R}. \tag{4}$$

b) Use the method of Frobenius to find series solutions for the differential equation:

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0 \tag{5}$$

in some interval $0 < x < R$. (Find the first 4 terms of each solution)