1)a) Find the general solution of $\frac{du}{dx} + \frac{1}{x}u = \frac{e^x}{x}$.

b) Solve the initial value problem

$$\frac{dy}{dx} + \frac{y}{4x} = \frac{e^x}{4xy^3},\ y(1) = e.$$

2) Consider the differential equation

$$y' = f(t, y) = \frac{-t + \sqrt{t^2 + 4y}}{2} \tag{1}$$

a) Find the region of continuity of $f(t, y)$ and $\frac{\partial f}{\partial y}(t, y)$ and sketch it in the $ty - plane$.

b) Verify that the functions $y_1(t) = 1 - t$ and $y_2(t) = \frac{-t^2}{4}$ are two solutions to the initial value problem

$$y' = f(t, y) = \frac{-t + \sqrt{t^2 + 4y}}{2},\ y(2) = -1. \tag{2}$$

Does this contradict the Existence Uniqueness Theorem? Why ?

c) Does the initial value problem

$$y' = f(t, y) = \frac{-t + \sqrt{t^2 + 4y}}{2},\ y(1) = 4 \tag{3}$$

have unique solution? why?

3) Consider the differential equation

$$(x + 2).sinydx + x.cosydy = 0. \tag{4}$$

a) Show that the equation (4) is not exact and make sure that the function $\mu(x, y) = xe^x$ works as integrating factor.

b) Find the general solution of equation (4).

4) Solve the differential equations:

a) $\sqrt{x^2 + 1}\frac{dy}{dx} = xy$ b) $y' = \frac{4x+3y}{2x-y}$.