1) Show that the following differential equation is exact and find its general solution:

$$(\frac{y}{x}+4x)+(lnx-3)\frac{dy}{dx}=0,\ x>0.$$

2) Solve the initial value problem:

$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\frac{dy}{dx}=0,\ y(0)=1, y'(0)=-1,\ y''(0)=1.$$

3) Consider the integro-differential equation:

$$\phi'(t)+\phi(t)=\int_0^t sin(t-s)\phi(s)ds,\ \phi(0)=1$$

.

- a)Solve the above equation by using Laplace transform.
- b)Transform the above integro-differential equation into an initial value problem by differentiating, then solve it and verify your solution in part (a).

4) a) Find the solution of the heat conduction problem:

$$u_{xx}=u_t,\ 0<x<40,\ t>0$$

with

$$u(0,t)=u(40.t)=0,\ t>0 \text{ and } u(x,0)=30,\ x>0.$$

b) Use Fourier coefficients in part (a) and Parseval's Theorem to find

$$\sum_{n=1,\ n\ odd}^{\infty}\frac{1}{n^2}=\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}.$$