Prince Sultan University

Department of Mathematical Sciences

MATH 223 – Third Examination

2 June 2008

Time allowed: 90 minutes　　　　　　　　　　Dr. Bahaa Eldin Abdalla
Maximum points: 40 points

1. (10 points) Let the vector space P_2 have the inner product

$$\langle \mathbf{p},\mathbf{q}\rangle = \int_0^1 p(x)q(x)\,dx$$

   (a) Find $\|\mathbf{p}\|$ for $\mathbf{p} = x^2 - 1$.

   (b) Find the cosine of the angle between $\mathbf{p} = x^2 - 1$ and $\mathbf{q} = x$.

   (c) Find $d(\mathbf{p},\mathbf{q})$ if $\mathbf{p} = x^2 + 1$ and $\mathbf{q} = x^2 - 2$.

2. (8 points) Find the eigenvalues and bases for the eigenspaces of A^{13} for

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

3. (4 points) Find a matrix P that diagonalizes $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, and determine $P^{-1}AP$.

4. (4 points) Determine whether the function $L : F(-\infty,\infty) \to F(-\infty,\infty)$, where $L(f(x)) = 1 + f(x)$ is a linear transformation. Justify your answer.

5. (6 points) Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2\}$ for R^2, where $\mathbf{v}_1 = (-2,1)$ and $\mathbf{v}_2 = (1,3)$, and let $L: R^2 \to R^3$ be the linear transformation such that $L(\mathbf{v}_1) = (-1,2,0)$ and $L(\mathbf{v}_2) = (0,-3,5)$. Find a formula for $L(x_1,x_2)$.

6. (8 points) Let $L : R^4 \to R^3$ be the function defined by $L(a_1,a_2,a_3,a_4) = (a_1 + a_2, a_3 + a_4, a_1 + a_3)$. Find a basis for range (L) and a basis for Ker(L).
