Prince Sultan University

MATH 221

Major Test II

Semester I, Term 161

Wednesday, December 21, 2016

Time Allowed: **90** minutes

Student Name: _____

Student ID #: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 6 problems, some with several parts. Make sure your paper has all these problems.

Question #	Max points	Student's Points
Q1	10	
Q2	10	
Q3	10	
Q4	8	
Q5	12	
Q6	10	
Total	**60**	

Q-1(10 points) Suppose that $f \in C^n[a,b]$ and $x_0, x_1, x_2, \ldots, x_n$ are distinct numbers in $[a,b]$. Then for some $\eta(x)$ in the interval (a,b), show that

$$f[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\eta(x))}{n!}$$

Q-2(10 Points) Show that the following function represents a cubic spline.

$$s(x) = \begin{cases} x & -1 \le x \le 0 \\ x + x^3 & 0 \le x \le 1 \\ 1 - 2x + 3x^3 & 1 \le x \le 4 \end{cases}$$

Q-3 (10 Points) Consider the points $x_0 = 0,\ x_1 = 0.4,$ and $x_2 = 0.7$ and for a function ***f(x),*** the divided differences are $f[x_2] = 6,\ f[x_1, x_2] = 10$ and $f[x_0, x_1, x_2] = \frac{50}{7}$. Use this information and construct the complete divided difference table for the given data points.

Q-4(8 points) Show that the local truncation error of the Trapoziodal rule to estimate definite integral $\int_a^b f(x)dx$ is $E_{T_1}(f) = -\frac{h^3}{12} f''(\eta(x))$, where $\eta(x) \in (a,b)$.

Q-5 (12 Points)

a) Let $f(x) = x^2 Cos(x)$. Compute the approximate value of $f''(1)$, taking $h=0.1$ and using central difference formula..

b) Compute the error bound for your approximation.

Q-6 (10 Points) Evaluate the integral $\int_0^1 e^{4x} dx$ using Simpson's 1/3 Rule with $n= 2$ and $n=4$.