Prince Sultan University

MATH 221

Final Exam

Semester I, Term 161

Tuesday, January 24, 2017

Instructor: Dr. Muhammad Dure Ahmad

Time Allowed: **3 hours**

Student Name: _____

Student ID #: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 9 problems, some with several parts. Make sure your paper has all these problems.

Question #	Max points	Student's Points
Q1	6	
Q2	8	
Q3	8	
Q4	6	
Q5	12	
Q6	12	
Q7	10	
Q8	8	
Q9	10	
Total	**80**	

Total/40 _____

Q-1(6 points) Use the ***fixed point theorem*** to check which of the following sequence will converge faster to $\sqrt{5}$ and why?

a) $x_{n+1} = x_n + 1 - \frac{x_n^2}{5}$

b) $x_{n+1} = \frac{1}{3}\left[3x_n + 1 - \frac{x_n^2}{5}\right]$

Q-2(8 Points) Use the ***Secant Method*** to find the solution accurate to 10^{-2} for the function

x - cos (x) =0, [0,π/2]

Take the initial approximation $x_0 = 0$ and $x_1 = \frac{\pi}{4}$

Q-3 (8 Points) Consider the following table

x	***0.1***	***0.2***	***0.4***	***0.5***
f(x)	***1.32295***	***1.67828***	***2.31315***	***2.42147***

Derive a third degree polynomial using ***Lagrange interpolation formula*** and find the approximation of ***f(0.3)***

Q-4: **(6 Points)** Determine $\|A\|_\infty$ for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 5 & -1 & 1 \end{bmatrix}$$

Q-5 (12 Points) Compute the next iteration x_3 using ***Muller's Method*** to approximate the roots of

$$f(x) = x^3 - 5x^2 + 4x$$

Take the initial approximation $x_0 = 4.4$, $x_1 = 5.2$ and $x_2 = 4.8$.

Q-6(12 points)

a) **Simple Trapezoidal rule** for solving the integral $I(f)=\int_a^b f(x)dx$ is given by

$$\int_a^b f(x)dx=\frac{b-a}{2}\left[f(a)+f(b)\right]$$

Show that the local **Error that Trapezoidal rule** makes in estimating the integral is

$$\int_a^b f(x)dx-\frac{b-a}{2}\left[f(a)+f(b)\right]=E_S(f)=-\frac{h^3}{12}f^{(3)}(\eta(x)),\ \text{where}\ \eta(x)\in(a,b)$$

b) .Use composite ***Simpson's 1/3 Rule*** to compute the integral $I(f)=\int_0^1 e^{4x}dx$, $n=8$

Q-7 (10Points) Consider the initial value problem $y' = x^3 + y, \quad y(0) = 2$. Compute *y(0.2)* and *y(0.4)* using ***Second order Runge Kutta Method***.

Q-8: **(8 Points)** Compute the first three steps to compute the numerically largest Eigen Value and corresponding Eigen Vectors of the following matrix using ***Power Method***. Take the initial approximation $X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

$$\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

Q-9: **(10 Points)** Show that the following system is diagonally dominant. Then compute the first two iterations to find the solution of the system of equation using ***Gauss-Seidle Method***

$$45x_1 + 2x_2 + 3x_3 = 58$$
$$-3x_1 + 22x_2 + 2x_3 = 47$$
$$5x_1 + x_2 + 20x_3 = 67$$

Take the initial guess (0,0,0).