Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

Numerical Analysis	MATH 221	Final Exam
Semester:	Spring Semester --Term 182	
Date:	Thursday April 20, 2019	
Time Allowed:	180 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Total	Total
Questions	1,2	3,4	5	6,7	8	9		
Max Marks	**16**	**16**	**12**	**14**	**12**	**10**	**80**	**40**

Q-1(8 points) : Show that the equation $x = N^{1/3}$ can be written as $x = Nx^{-2}$ and the associated iterated scheme $x_{n+1} = Nx_n^{-2}$, $n \geq 0$ will not be successful in finding the third root of the positive number *N*. Completely justify your answer.

Q-2(8 points) Perform two iterations of the Newton's method to find the root of the equation $f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}$. Take the initial approximation $x_0 = 0.05$. Also compute relative error for each iteration.

Q-3(8 points) Use composite Simpson'1/3 Rule to evaluate the integral $\int_{-1}^{3} f(x)dx$ for the following data with the spacing *h=0.5*

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
f(x)	7	5	3.5	4	5.5	6	6.5	5	4.5

Q-4(8 points) Let $f(x) = ln(x^2 + 1)$, compute the value of $f[2, -1, 2, -1]$

Q-5: (12 points) Find the Hermite interpolating polynomial of degree 3 for the following data. Simplify your answer**.**

	$f(x)$	$f'(x)$
$x_0 = 1$	$f_0 = 1$	$f'_0 = 1/2$
$x_1 = 4$	$f_1 = 2$	$f'_1 = 1/4$

Q-6(8 points) Compute *y(1.1)* using ***Taylor's Series method of order 2*** to solve initial value problem $\frac{dy}{dx} = xy^2 + e^x$, ***y(1)=4.***

Q-7: (6 points) Let $x \in \mathbb{R}^n$, then by using the definition of L_2 and L_∞ norms, show that

$$\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty$$

Q-8(12 points) Rearrange the following system of equations so that it becomes diagonally dominant. Write the system in the form $X^{(k+1)} = HX^{(k)} + C$. Clearly identify what is *H* and *C*. Then use three iterations of the **Gauss-Seidle Method** to approximate the solution

$$\mathbf{3x - 6y + 2z = 23}$$
$$\mathbf{-4x + y - z = -15}$$
$$\mathbf{x - 3y + 7z = 16}$$

Take the initial approximation (1,1,1). Compute the relative error in each step.

Q-9(10 points) Perform 6 iterations to find the numerically largest Eigen value and corresponding Eigen vector of the following matrix using Power Method. Take the initial vector x_0= [*1, 1, 1*]^T.

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

What can you guess about the spectral radius $\rho(A)$?