Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences

COURSE DETAILS:

Numerical Analysis MATH 221 MAJOR EXAM II	
Semester:	Spring Semester --Term 182
Date:	Tuesday April 02, 2019
Time Allowed:	90 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Total
Questions					
Marks	16	16	10	8	50

Q-1(8 points) Let $f(x) = \frac{1}{x}$, using the points a, b and c, show that $f[a, b, c] = \frac{1}{abc}$.

Q-2(8 Points) Find the linear spline that interpolates the following data

x	1	2	3	4
f(x)	1	2/3	1/2	2/5

Then find the value of ***f(1.7)***

Q-3 (8+8 Points) a) Compute the error for the three points formula of second derivative

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

b) Let the velocity of a rocket is given by

$$v(t) = 2000 \ln\left[\frac{14\times 10^4}{14\times 10^4 - 2100t}\right] - 9.8t, \qquad 0 \le t \le 30,$$

Find its rate of change of acceleration (that is $\boldsymbol{v''(t)}$) at *t=16 sec* using the above formula, with step size of 2 seconds.

Q-4 (10 Points) Consider the points $x_0 = 1,\ x_1 = 5,$ and $x_2 = 7$ and for a function *f(x)*, the divided differences are $f[x_2] = 15,\ f[x_1, x_2] = 10$ and $f[x_0, x_1, x_2] = 20$. Use this information and construct the complete divided difference table for the given data points.

Q-5(8 points) Use Simpson's1/3 rule to evaluate the integral $\int_0^1 \frac{dx}{x^2+6x+10}$, using 4 subintervals.

Formula Sheet

Hermite Interpolating Polynomial

Suppose that *f(t)* is continuously differentiable on [*a*, *b*] and the numbers $x_0, x_1, \cdots, x_n \in [a,b]$ are unique, and let $L_{nj}(x)$ be the Lagrange interpolation function. Then

$$P(x) = H_{2n+1}(x) = \sum_{j=0}^{n} f_j H_{nj}(x) + \sum_{n=0}^{j} f'_j \hat{H}_{nj}(x)$$

Where

$$H_{nj}(x) = \left[1 - 2(x - x_j)L'_{nj}(x_j)\right]\left(L_{nj}(x)\right)^2$$
$$\hat{H}_{nj}(x) = (x - x_j)\left(L_{nj}(x)\right)^2$$

Linear Spline Formula:

$$s_k(x) = y_k + \frac{y_{k+1} - y_k}{x_{k+1} - x_k}(x - x_k)$$

Derivatives Formula:

Backward Difference:

1: First Order:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$$
$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$$

2: Secod Order

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$$
$$f''(x_i) = \frac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})}{h^2}$$

Forward Difference:

1: First Order:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$
$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

2: Secod Order

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$$
$$f''(x_i) = \frac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2}$$

Central Difference:

Secod Order:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$
$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

Integration Formula:

Simpson's 1/3 Rule:

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right],$$ **Absolute Error:** $$\frac{h^5}{90} f^{(4)}(\eta(x)),\ a \le \eta \le b,\ \text{where } h = \frac{b-a}{2}$$

Composite Simpson's 1/3 rule:

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0) + 4\left(f(x_1) + f(x_3) + \cdots + f(x_{2N-1})\right) + 2\left(f(x_2) + f(x_4) + \cdots + f(x_{2N-2})\right) + f(x_{2N})\right]$$

Absolute Error: $$\frac{(b-a)h^4}{180} f^{(4)}(\eta(x)),\ a \le \eta \le b,\ \text{where } h = \frac{b-a}{2N}$$