Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences

COURSE DETAILS:

Numerical Analysis	MATH 221 MAJOR EXAM I
Semester:	Spring Semester --Term 182
Date:	Tuesday February 26, 2018
Time Allowed:	90 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

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Questions					
Marks	16	16	8	10	50

Q-1(8 points) Use bisection method to find c_1, c_2, c_3, c_4 and c_5 in order to compute $\sqrt[5]{30}$ using the interval [1.8, 2.4]. Also compute the error bound and exact error of your approximation.

Q-2(8 Points) Let $\boldsymbol{f(x) = e^{4x}}$. Estimate $f(x_T) - f(x_A)$, where x_T represents exact value and x_A represents approximate value. Then by using Mean Value theorem, show that

$$\boldsymbol{Rel\big(f(x_A)\big) = 4Abs(x_A).}$$

Here ***Rel*** stands for relative error and ***Abs*** stands for Absolute error.

Q-3 (8+8 Points) a) Show that the equation $x^3 - 4x^2 + 6 = 0$ can be written in the fixed point form as $x_{n+1} = \left(4 - \frac{6}{x_n^2}\right)$. Then use fixed point theorem to show that there exists a unique fixed point in the interval [3,4]. **Completely justify your answer.**

b) Estimate the number of iterations required in fixed point method for the above problem to obtain an accuracy of 10^{-3}.

Q-4 (8 Points) Perform three iterations of Secant method to find roots of $f(x) = x^3 - 3x + 1 = 0$. Take the initial approximation $x_0 = 1$ and $x_1 = 0.5$.

Q-5(10 points) Find two approximation x_1 and x_2 to a zero of $P(x) = x^3 - 7x^2 + x - 15$ using Newton's method with $x_0=7$ and Horner's method to find $P(7)$ and $P'(7)$.

Formula Sheet

Secant Method:

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1}) f(x_n)}{f(x_n) - f(x_{n-1})} \ , \quad n \geq 0$$

Muller's Method:

$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$, $\quad b = \frac{f_1 - f_0 - a h_1^2}{h_1}$ and $X_{new} = X_0 - \frac{2C}{b \pm \sqrt{b^2 - 4ac}}$

Where $f_0 = f(x_0) = C$, $f_1 = f(x_1)$, $f_2 = f(x_2)$,

$h_1 = x_1 - x_0$, $h_2 = x_0 - x_2$

Atkinson's method: $\tilde{X}_n = X_n - \frac{\left(X_{n+1} - X_n\right)^2}{X_{n+2} - 2X_{n+1} + X_n}$