Prince Sultan University

Deanship of Educational Services
Department of Mathematics
and General Sciences

COURSE DETAILS:

Numerical Analysis	MATH 221	Final Exam
Semester:	Spring Semester --Term 181	
Date:	Tuesday December 18, 2018	
Time Allowed:	180 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Page 5	Page 6	Total	Total
Questions								
Max Marks	**14**	**10**	**16**	**14**	**12**	**14**	**80**	**40**

Q-1(8 points) Let α_1 and α_2 be two fixed point of the quadratic function $f(x) = \frac{1}{2}x^2 - \frac{3}{2}x + 2$

a) Find the values of both fixed points
b) For which point the fixed point method will converge

Q-2(6 points) The iterative scheme $X_{n+1} = 2 - (1 + a)X_n + aX_n^2, \quad n \geq 0$, converges to $\propto = 1$ for some value of '***a'.*** Find the value of ***a*** for which the convergence is at least quadratic.

Q-3(10 points) Do two steps of Muller's method to find the root of $f(x) = x^3 + 2x - 1 = 0$, using the initial values $x_0 = 1$, $x_1 = 1.5$, $x_2 = 0.5$. ***(Hint: In the second step, replace x_1=1.5 with the new value and use new values x_0=0.5, x_1=1 and x_2=x_{new}).***

Q-4(8 points) Use **Simpson's $\frac{1}{3}$ rule** to evaluate the integral $\int_0^2 \frac{e^{x^2}}{1+x^2} dx$ with 4 **subintervals**.

Q-5(8- points) Find the linear Spline that interpolates the following data points. Also compute the value of ***f(-0.5)***

x	***-2***	***-1***	***1***	***2***
f(x)	***1***	***0.75***	***0.5***	***0.4***

Q-6 (6 points) Let $f(x) = x - e^{-x}$. Find the value of $f[0,1,0,1]$

Q-7(8 points) Consider the following differential equation

$$\frac{dx}{dt} = -2.2067 \times 10^{-12}(x^4 - 81 \times 10^8), \qquad x(0) = 1200$$

Find the value at $t = 480$ seconds using **Runge-Kutta 2nd order method**. Assume a step size of $h = 240$ seconds.

Q-8(12 points) Show that the following system is diagonally dominant. Then use two iterations of the **Gauss-Seidle Method** to approximate the solution

$$12x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76$$

Take the initial approximation$(1,0,1)$. Also compute the absolute relative error at the end of the each iteration.

Q-9(8points) Compute the ***first four steps*** to approximate the spectral radius of the following matrix using Power Method. Take the initial approximation $X_0 = (1,1,1)^T$

$$\begin{bmatrix} 4 & 10 & 0 \\ -1 & 3 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

Q-10 (6 points) Determine $\|A\|_\infty$ for the matrix $A = \begin{bmatrix} 1 & 3 & -4 & 5 \\ 0 & -2 & 5 & 1 \\ -4 & 3 & 2 & 0 \\ 2 & -1 & 3 & -5 \end{bmatrix}$