Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences

COURSE DETAILS:

Numerical Analysis	MATH 221	MAJOR EXAM 2
Semester:	Spring Semester --Term 181	
Date:	Tuesday November 27, 2018	
Time Allowed:	90 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Total
Questions					
Marks	12	14	12	12	50

Q-1(12 points) : Consider the following table having data for $f(x) = e^{3x} \cos\ 2x$

x	0.1	0.2	0.4	0.5
$f(x)$	1.32295	1.67828	2.31315	2.42147

Find the approximation of *f(0.3)* using suitable Lagrange interpolation formulae. Also estimate an error bound for the approximation.

Q-2(6 Points) Use Composite Trapezoidal rule for $n=4$ to compute the integral $\int_0^1 \sqrt{1+4x^2 e^{2x^2}}\,dx$.

Q-3 (8 Points) Determine the value of step size h and the number of subintervals N to approximate the integral $I = \int_0^2 \frac{dx}{4+x}$ to an accuracy of 10^{-5} Using <u>Simpson's Rule.</u> **<u>Do not Compute the integral.</u>**

Q-4 (4+4+4 Points) a) Use the method of undermined co-efficient to derive forward difference formula for $f''(x)$ in the form

$$f''(x) = Af(x+2h) + Bf(x+h) + Cf(x)$$

b) Use this formula to compute $f''(1.5)$ for the function $f(x) = \frac{x+e^{x^3}}{1+x}$, *using* $h = 0.01$

c) Compute the error bound for the formula derived in part (a)

Q-5(12 points) If *f(x)=p(x)q(x),* then show that

$$f[x_0, x_1] == p[x_1]q[x_0, x_1] + q[x_0]p[x_0, x_1]$$

Also find the values of *p[0,1] and q[0,1]* when *f[0,1]=4, f(1)=5 and p(1)=q(0)=2.*

Formula Sheet

Hermite Interpolating Polynomial

Suppose that *f(t)* is continuously differentiable on [*a*, *b*] and the numbers $x_0, x_1, \cdots, x_n \in [a,b]$ are unique, and let $L_{nj}(x)$ be the Lagrange interpolation function. Then

$$P(x) = H_{2n+1}(x) = \sum_{j=0}^{n} f_j H_{nj}(x) + \sum_{j=0}^{n} f_j' \hat{H}_{nj}(x)$$

Where

$$H_{nj}(x) = \left[1 - 2(x - x_j) L_{nj}'(x_j)\right]\left(L_{nj}(x)\right)^2$$

$$\hat{H}_{nj}(x) = (x - x_j)\left(L_{nj}(x)\right)^2$$

Linear Spline Formula:

$$s_k(x) = y_k + \frac{y_{k+1} - y_k}{x_{k+1} - x_k}(x - x_k)$$

Derivatives Formula:

Backward Difference:

1: First Order:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$$

$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$$

2: Secod Order

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$$

$$f''(x_i) = \frac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})}{h^2}$$

Forward Difference:

1: First Order:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

2: Secod Order

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$$

$$f''(x_i) = \frac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2}$$

Central Difference:

Secod Order:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

Integration Formula:

Simpson's 1/3 Rule:

$\int_a^b f(x)dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$, **Absolute Error:** $\frac{h^5}{90} f^{(4)}(\eta(x)),\ a \le \eta \le b,\ \text{where } h = \frac{b-a}{2}$

Composite Simpson's 1/3 rule:

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0) + 4\left(f(x_1) + f(x_3) + \cdots + f(x_{2N-1})\right) + 2\left(f(x_2) + f(x_4) + \cdots + f(x_{2N-2})\right) + f(x_{2N})\right]$$

Absolute Error: $\frac{(b-a)h^4}{180} f^{(4)}(\eta(x)),\ a \le \eta \le b,\ \text{where } h = \frac{b-a}{2N}$