Prince Sultan University

Deanship of Educational Services
Department of Mathematics and General Sciences

COURSE DETAILS:

Numerical Analysis MATH 221 MAJOR EXAM I	
Semester:	Spring Semester --Term 181
Date:	Tuesday October 23, 2018
Time Allowed:	90 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

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Questions					
Marks	16	12	12	10	50

Q-1(16 points) Consider the equation $x^3+4x^2-10=0$. Show that this can be written in the two forms. Then use the fixed point theorem to check which one will converge to a unique fixed point in the interval $[1, 2]$. Justify your answer. Then compute four iterations to check your findings.

a) $x = g_1(x) = \left(\frac{10}{x} - 4x\right)^{1/2}$

b) $x = g_2(x) = \left(\frac{10}{4+x}\right)^{1/2}$

Q-2(6 Points) Compute the first two terms (i.e., $\widehat{p_1}\ and\ \widehat{p_2}$) of the sequence $p_n = 3^{-p_{n-1}},\ n \geq 1$ Using Atkin's method. Take $p_0 = 0.5$.

Q-3 (6 Points) Consider the mathematical problem P defined by z = x + y. In order to examine how errors in x and y propagate to z. We define our approximations by $\Delta x = x - \hat{x}$, and $\Delta y = y - \hat{y}$, where Δx and Δy are errors in x and y respectively and $\hat{x}$ and $\hat{y}$ are approximated values of x and y respectively. Compute the condition number with respect to relative error using 1-norm.

Q-4 (12 Points) Show that the iterative procedure for evaluating the reciprocal of a number using secant method can be written as

$$X_{n+1} = X_n + X_{n-1}(1 - NX_n)$$

Then use this scheme to find the reciprocal of $N=8$, using $X_0=0.05$ and $X_1=0.1$.

Q-5(10 points) Consider a non-linear function given by

$$F = \frac{A^2\sqrt{B}}{C^3}$$

With the values
$A = 1.214 \pm 0.005$
$B = 5.1 \pm 0.02$ and
$C = 4.29 \pm 0.015$

Compute the absolute maximum propagation error in F.

Formula Sheet

Secant Method:

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1}) f(x_n)}{f(x_n) - f(x_{n-1})} \ , \quad n \geq 0$$

Muller's Method:

$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$, $\quad b = \frac{f_1 - f_0 - a h_1^2}{h_1}$ and $X_{new} = X_0 - \frac{2C}{b \pm \sqrt{b^2 - 4ac}}$

Where $\quad f_0 = f(x_0) = C \ , \quad f_1 = f(x_1) \ , \quad f_2 = f(x_2) \ ,$

$h_1 = x_1 - x_0 \ , \quad h_2 = x_0 - x_2$

Atkinson's method: $\tilde{X}_n = X_n - \frac{\left(X_{n+1} - X_n\right)^2}{X_{n+2} - 2X_{n+1} + X_n}$