Prince Sultan University

Deanship of Educational Services
Department of Mathematics
and General Sciences

COURSE DETAILS:

Numerical Analysis	MATH 221 Final Exam
Semester:	Spring Semester --Term 172
Date:	Thursday May 10, 2018
Time Allowed:	180 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

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Questions								
Max Marks	**16**	**16**	**14**	**12**	**10**	**12**	**80**	**40**

Q-1(8 points) Develop an iterative scheme for evaluating the root of a number using **Newton's method** (i.e., you need to find a root of $X = \sqrt[p]{N}$, where p is a positive integer). Use this scheme to approximate $\sqrt[3]{11}$. Take the initial guess $X_0 = 2$.

Q-2(8 points) Generate the first three terms $\widetilde{X_1}$, $\widetilde{X_2}$ and $\widetilde{X_3}$ of the sequence

$X_n = \sqrt{\frac{e^{X_{n-1}}}{3}}$, with $X_0 = 0.75$ using **Atkin's Method**.

Q-3(8 points) Consider the Mathematical Problem defined by $= \frac{x}{y}$. Define the approximation $\Delta x = x - \hat{x}$ and $\Delta y = y - \hat{y}$, where $\hat{x}$ and $\hat{y}$ are approximate values of *x* and *y* respectively and Δx and Δy are error terms in *x* and *y*. Show that

$$Rel(z) \le Rel(x) + Rel(y)$$

Where $Rel(x), Rel(y) and\ Rel(z)$ are relative errors in the *x*, *y* and *z* values respectively.

Q-4(8 points) Use **Simpson's $\frac{1}{3}$ rule** to evaluate the integral $\int_0^1 \frac{dx}{x^2+6x+10}$ with **4 subintervals**.

Q-5(8+6 points) **a)** Suppose that $f \in C^n[a,b]$ and $x_0, x_1, \cdots, x_n$ are distinct numbers in $[a,b]$. Then for some points $\eta(x)$ on the interval (a,b) spanned by $x_0, x_1, \cdots, x_n$, show that

$$\boldsymbol{f[x_0, x_1, \cdots, x_n] = \frac{f^{(n)}(\eta(x))}{n!}}$$

b) Let $f(x) = e^{-x}$. Then using the result of part (a), find the value of $f[0,0,1,1]$

Q-6(12 points) A metal ball at 1200 ℃ is allowed to cool down in air at an ambient temperature of 300 ℃. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

$$\frac{d\theta}{dt} = -2.2067 \times 10^{-12}(\theta^4 - 81 \times 10^8), \qquad \theta(0) = 1200$$

where θ is in ℃ and t in seconds. Find the temperature at $t = 480$ seconds using **Runge-Kutta 2nd order method**. Assume a step size of $h = 240$ seconds.

Q-7(10 points) Show that the following system is diagonally dominant. Then use three iterations of the **Gauss-Seidle Method** to approximate the solution

$$-4x_1 + x_2 + x_3 = -8$$
$$3x_1 - 6x_2 + 2x_3 = 23$$
$$x_1 - 3x_2 + 7x_3 = 17$$

Take the initial approximation $(0.9, -3.1, 0.9)$**.**

Q-8(12 points) Compute the l_2-norm of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$, **using Power method**.

Take $(1,1,1)$ as initial approximation for the Eigen vector. (Just compute 4 iterations)

(Recall that $\|A\|_2 = \sqrt{\rho(A^T A)}$, where ρ represents the spectral radius).

Scratch Paper