

Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

Numerical Analysis		MATH 221	MAJOR EXAM 2
Semester:	Spring Semester --Term 172		
Date:	Wednesday April 18, 2018		
Time Allowed:	90 minutes		

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Total	Total
Questions					
Marks	15	15	15	45	15

Q-1: (8 points) Consider the points $x_0 = 1$, $x_1 = 3$ and $x_3 = 6$. And for the function $f(x)$, the divided differences are $f[x_2] = 3$, $f[x_1, x_2] = 10$ and $f[x_0, x_1, x_2] = 15$. Use this information to construct the complete divided difference table.

Q-2: (7 points) The function $f(x)$ satisfied the equation $f'''(x) = x^2 f(x)$ and the condition $f(0) = 3$, $f(0.1) = 5$. Use the central difference formula for $f''(x)$ and step size $h=0.05$ to estimate the value of $f(0.05)$.

Q-3: (8 points) Use Simpson rule for $n=6$ to compute the integral $\int_0^2 \sqrt{1 + 4x^2 e^{2x^2}} dx$

Q-4: (7 points) Consider the initial value problem

$$\frac{dy}{dx} = \frac{t^2 + y^2}{2}, \quad y(0) = 0$$
$$0 \leq t \leq 1, -1 \leq y(t) \leq 1$$

Determine how small the step size should be that the Global error does not exceed to 10^{-6}

Q-5: (9+6 points) a) Show that the error bound for linear Lagrange Interpolation passing through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ is $\frac{M(x_1-x_0)^2}{8}$, where $M = \max_{x_0 \leq \eta \leq x_1} f''(\eta)$.

b) Use the error bound in the approximation of Linear Lagrange polynomial for the following data points obtained from the function $f(x)=\ln(x+2)$.

x	0	2	4
$f(x)$	0.6931	1.3862	1.7918